LILONGWE UNIVERSITY OF AGRICULTURE AND NATURAL RESOURCES

DEPARTMENT OF BASIC SCIENCES
PHYSICS:MECHANICS

PHY 111

MODULE 1

MODULE WRITERS

Physics: Mechanics

PHY111

Module 1

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MODULE OVERVIEW

Physics module one is a composition of fundamental principals in mechanics. It covers the key concepts in fundamental quantities and units, vectors, basic mechanic principals, motion and acceleration, work, energy and power and simple machines. These six areas which have been distributed into units. Each unit has simplified illustrations, examples, activities, test and suggested solutions to the activities and test. At the end of the module there is a module test to be attempted by all students.

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UNIT 1: FUNDAMENTAL QUANTITIES AND UNITS

1.0 Introduction

Measurements are part and parcel of everyday life. You may have weighed masses and volume of different substances e.g. 1kg of sugar, 500g of flour, 2litres of water. All substances that you are able to measure are called physical quantities.

In most cases measurements that we take are associated with errors. In actual experiments it is very difficult to record 100% accurate measurements. This is the case due errors that may be associated either with you as a researcher or the instruments being used. All measurents of a physical quantity that you take in science are recorded in SI units.

Physics is a branch of science that deals with measurements. It involves scientific procedures and processes that deal with error analysis in measurements and standard presentation of figures in science. Your understanding of this unit is very important to both Physics and General Science.

1.1. Objectives

By the end of this unit you should be able to:

- Define physical quantities
- Differentiate derived quantities from basic quantities
- Analyze dimensions of different physical quantities
- Evaluate errors associated physical quantities

Key terms

Ensure that you understand the key terms or phrases used in this unit as listed below.

- **Physical quantity**
- **Basic Units**
- **Derived Units**

- **Dimensions**
- **Precision**
- **Accuracy**

1.2. Physical Quantities

A physical quantity is a quantity that can be measured. In you homes, schools and in everyday life you take measurements of maize flour, sugar, water, paraffin, etc. All these are examples of physical quantities. We can classify physical quantities in two, scalar quantities and vector quantities.

1.2.1 **Scalar Quantities**

Quantities that have magnitude only, e.g. mass, temperature, time, length. The magnitude of a scalar quantity indicates its numerical value and unit of measure.

1.2.2 **Vector Quantities**

Quantities that have both magnitude and direction, e.g. velocity, force, momentum, displacement, etc.

The magnitude of a vector is a scalar that indicates only how large or small the vector is.

Activity **1**.

After we have discussed physical quantities, you are supposed to record measurements of some substances as stipulated in physics practical module, unit 1.

1.3 Units

A quantity used as a standard of measurement is called a unit. In Physics we use two sets of units, thus basic units and derived units.

1.3.1 Basic **Units**

Basic units are units selected for measuring mass, length and time. These quantities cannot be expressed in terms of other quantities. We have seven basic quantities in physics. The units of all the other physical quantities in both science and engineering are derived from these seven basic quantities.

Table 1: Seven Basic Quantities

QUANTITY	**SYMBOL**	**UNIT NAME**	**UNIT SYMBOL**
Length	L, *l*	metre	m
Mass	*M*	kiliogram	kg
Time	*t*	second	s
Electric current	I	ampere	A
Temperature	T	Kelvin	K
Luminous intensity	I	candela	cd
Quantity of matter		mole	mol

1.3.2 Derived **Units**

Having looked at basic or fundamental units now we want to consider a combination of several basic units. Units obtained from a combination of several basic units through multiplication and division are referred to as derived units.

You may alternatively define derived units as units of physical quantities that can be expressed in terms of fundamental units. We normally use units of length, mass and time to come up with derived units.

We have plenty of examples of derived units that we encounter in everyday life for example, velocity, force, work, etc.

Example 1, Show that the unit of force is a derived quantity

Solution: $velocity = \frac{displacement}{time} = \frac{length}{time} = \frac{m}{s}$

1.3.3 Multiples and sub-multiples of SI Units

We can generate multiples and sub-multiples of SI units by adding appropriate prefixes to the units. Every prefix has a numerical value, for instance we use kilo to represent 1000, micro to represents 10^{-6}, i.e. 20 kilowatts = 20 x 1000 watts = 20 000 watts.

Sometimes prefixes may be represented using a factor or symbol as shown in table 2.

Table 2: Multiples and sub-multiples of SI units

FACTOR	PREFIX	SYMBOL
10^{24}	yotta -	Y
10^{21}	Zetta -	Z
10^{18}	Exa -	E
10^{15}	Peta -	P
10^{12}	tera -	T
10^{9}	giga -	G
10^{6}	mega -	M
10^{3}	kilo -	k
10^{2}	hecto -	h
10^{1}	deca -	da
10^{-1}	deci -	d
10^{-2}	centi -	c
10^{-3}	milli -	m
10^{-6}	micro -	µ
10^{-9}	nano -	n
10^{-12}	pico -	p
10^{-15}	femto -	f
10^{-18}	atto -	a
10^{-21}	zepto -	z
10^{-24}	yokto -	y

As you may see from table 2, multiples and submultiples are used to shorten long numbers e.g. we can 20 000 000 000 bites as 20 x 10^{9} = 20gigabites =20 GB.

1.3.4 Conversions

After looking at multiples and sub-multiples, we can now convert SI units from one unit to another

We use several conversion factors to translate a measurement from one unit to another unit. In table 4, we are translating units from column A to B by multiplying a unit in column B with a factor. For instance to change 1mile into a meter, you have to multiply a meter with a factor of 1609.344.

In cases where a unit cannot be translated directly to other unit, we translate it first to intermediate unit/s and then proceed to find our desired unit.

Table 3: Unit Convertion

UNIT A	UNIT B
1 Mile	1609.344 meter
1 Inch (in)	0.0254 meter
1 foot (ft)	12 inches
1 foot	0.3048 meter
1 m/s	3.6 km/hr
1 mile/h	0.447 m/s

Activity 2

1. State one difference between basic units and derived units
2. Write the following figures using prefixes
   a. 500 000 000 000 000 bites
   b. 8.0 x 10^{-20} metres
3. Convert the following units
   a. 40cm to inches
   b. 30 miles/hour to metres/ second

1.4 Dimensions and Dimensional Analysis

A dimension is a physical variable used to specify the behavior or nature of a particular system. E.g. the length of a rod is so many meters

We use square brackets [ ], to denote dimensions. For example we can represent the dimension of velocity as

$$[v] = \frac{[L]}{[T]}$$

So far we have defined a dimension, but in most cases we use dimensions to express a dimension. The dimensions of a physical quantity are the powers to which the basic quantities (M, L and T) must be raised to represent a physical quantity. We represent a basic quantity, which does not appear in the physical quantity by raising it to power zero.

Example 2.

Find the dimension of acceleration *a*, and the dimensions of M, L and T in the dimensional formula of acceleration.

The dimension of acceleration; $\left[a\right]=\frac{\left[M^0LT^{-1}\right]}{T}=\left[M^0LT^{-2}\right]$

Hence the dimensions of acceleration are 0 in mass, 1 in length and -2 in time. Alternatively we can present dimensions of acceleration as M = 0, L = 1 and T = -2.

1.4.1 Dimensional Analysis

Dimensional Formula

This is the expression of a physical quantity in terms of its dimensions. For example, the dimensional formula for force is

$$\left[F\right]=\left[MLT^{-2}\right]$$

Dimensional Equation

If you express an equation containing physical quantities, each quantity represented by its dimension formula, the resulting equation is referred to as Dimensional Equation.

Consider the formula $v=u+at$

Here *u* is initial velocity of the body, *a* is acceleration and *t* is the time taken to attain final velocity *v*. Writing this equation in the dimensional form, we have

$$\left[M^0LT^{-1}\right]=\left[M^0LT^{-1}\right]+\left[M^0LT^{-1}\right]+\left[M^0L^0T\right]$$

NOTE: in a dimensional equation, the dimensions in every term should be the same

TABLE 3. Dimensional formulae of Physical Quantities

No.	**Physical Quantity**	**Relation with other physical quantities**	**Dimensional Formula**	**SI Unit**
1.	Area	Length x breadth	[L] [L] = $[L^2]$	m^2
2.	Density	Mass ÷ Volume	[M]÷ $[L^3]$ = $[ML^{-3}]$	Kg/m^2
3.	Work	Force x distance	$[MLT^{-2}][L]= [ML^2T^{-2}]$	Nm

1.4.2 Categories of Physical Quantities

When you carefully study dimensional formula of the physical quantities you will discover that there are four categories of physical quantities.

a. **Dimensional variables:** Physical quantities that have dimensions but do not have fixed value e.g. force, velocity, work, power, etc.

b. **Dimensional constants**: Physical quantities which posses dimensions and have fixed value e.g. gravitational constant, Planck's constant, etc.

c. **Dimensionless variables**: Physical quantities, which have neither dimensions nor fixed value e.g.
   - **Unfixed Constants:** specific gravity, strain,
   - **Trigonometric functions:** Cos 30°
   - **Logarithmic functions and exponential functions**: log 10,

d. **Dimensionless constants**: Physical quantities, which do not posses dimensions but have a fixed value e.g.
   - Numerical values: 1, 2, 200
   - Fixed constants: π, index of refraction (n), etc.

1.4.3 Uses of Dimensional Formula (Analysis)

Dimensional analysis is used in the following ways:

- To check the correctness of an equation
- To remember a forgotten formula

- To derive relationship between different physical quantities
- To convert one system of units to another
- To find the dimensions of constants in a given relation

CHECKING THE CORRECTNESS OF A PHYSICAL RELATION (EQUATION)

In order to check if a physical relation or a scientific equation is correct or not, dimensional analysis is applied. When the dimensions of the two sides of the equation thus, the right hand side (RHS) and the left hand side (LHS) are equal a scientific equation is considered as dimensionally correct and visa versa.

Example 3

Show if the equation, $\mathbf{v = u+at^2}$ is dimensionary correct or not. Where **v** is final speed, **u** is initial speed, **a** is acceleration and t is time.

Solution: $[v]=\frac{L}{T}$, $[u]=\frac{L}{T}$, $[a]=\frac{L}{T^2}$ and $[t^2]=T^2$

Then $v=u+at^2$

$$L.H.S=[v]$$
$$R.H.S=[u]+[a][t^2]$$

$$L.H.S=\frac{L}{T}$$
$$R.H.S=\frac{L}{T}+\frac{L}{\cancel{T^2}}\times\cancel{T^2}$$

$$=\frac{L}{T}+L$$

L.H.S = $\frac{L}{T}$ and R.H.S = $\frac{L}{T}+L$

$$\therefore \frac{L}{T}\neq\frac{L}{T}+L$$

L.H.S ≠ R.H.S

The equation is dimensionally incorrect.

TO CALCULATE DIMENSIONS OF UNKNOWN QUANTITY

When you want to find an unknown variable in an equation, you simply make it the subject of the formula and evaluate dimensions of the terms on the right hand side of the equation.

Example 4

A sphere of radius *a*, moves with velocity *v*, in a medium and the force F acting on it, is given as;

$$F = 6\pi\eta av$$

Find the dimensions of η .

Solution: You first have to make η subject of the formula.

$$\eta = \frac{F}{6\pi av}$$

Then, work out dimension of each quantity

$\left[F\right] = \frac{ML}{T^2}$, $\left[a\right] = L$, $\left[v\right] = \frac{L}{T}$, 6 and π are dimensionless

Note that we can represent $\frac{ML}{T^2}$ as MLT^{-2} and $\frac{L}{T}$ as LT^{-1}

$$\eta = \frac{MLT^{-2}}{L \times LT^{-1}}$$

$$= \frac{MLT^{-1} \times T^{-1}}{L \times L \times T^{-1}}$$

$$= \frac{MT^{-1}}{L} = ML^{-1}T^{-1}$$

$$\therefore \quad \eta = ML^{-1}T^{-1}$$

In this case we have found that dimensions of η are $ML^{-1}T^{-1}$.

As you have observed from example 2, it is easier to work out SI units for an unknown quantity using dimensional analysis. Having found the dimensions of unknown quantity we can proceed to find SI units for unknown quantity. Each dimension gives a unit.

Consider calculating SI units for $\eta = ML^{-1}T^{-1}$

Then M = kg, $L^{-1} = m^{-1}$, $T^{-1} = s^{-1}$

$\therefore$ SI units for $\eta = kgm^{-1}s^{-1}$

Activity 3

1. What are the dimensions of length and time in the dimensional formula of power?

2. If F is the electric force, B = magnetic field, I = current and l = length of a conductor, find the dimensions and SI units of B in the equation,

$$F = BIl$$

1.5 Measurements and Accuracy

1.5.1 Measurements

Measurement is the comparison of unknown physical quantity with a known fixed quantity.

For example if the length of the rod is 3 metres, it means the unit of length is the metre and this unit is contained 3 times in the length of that rod.

Two things represent the magnitude of a physical quantity;

i. The unit in which the quantity is measured e.g. metre

ii. The numerical value – this expresses how many times a unit is contained in a physical quantity, e.g. 3.

1.5.2 Accuracy

You may recall from your secondary school physical science that accuracy is defined as the measure of how close the measured value of a quantity corresponds to its true value. Oftenly we use the term accuracy hand in hand with precision. The degree of reproducibility or the closeness between repeated measurements is called precision.

Every measuring instrument has an inherent limit of precision or accuracy. This limit is determined by the list count of the instruments. No matter how carefully we measure, we can never obtain a result more precise than the limit of our measuring device. In experiments we calculate the limit of precision by considering ½ of the smallest division built into the device.

When you are using a vernier calliper that has a smallest unit of measurement as 0.1mm, its limiting precision is 0.1 x ½ = ± 0.05mm = ± 0.005 cm. Suppose using this vernier calliper, you have measured the length of a rod as 5.95cm, your limit of precision will be the same ± 0.05cm. The length of the lord will be represented as 5.95±0.05 cm. This means the true value of the length lies between 5.95 – 0.05 cm (5.9 cm) and 5.95+0.05 cm (6.0 cm).

Significant figures.

The number of digits in a measurement about which we are certain, plus one additional digit which is uncertain are known as significant figures. The greater the number of significant figures obtained in a measurent, the more accurate is the measurement. But in most cases the number of significant figures is limited by a limit of precision.

Rules for significant figures

When we are using significant figures we consider the following rules:

i. All non-zero digits are significant e.g. 157.23 g contains five significant figures
ii. All zeros between two nonzero digits are significant e.g. 305.008 m contains six significant figures

iii. Unless stated otherwise, all zeros to the left of an **understood** decimal point but to the right of a nonzero digit are not significant e.g. 904,000s contains three significant figures.

iv. All zeros to the left of an **expressed** decimal point and to the right of a nonzero digit are significant e.g. 406.000 contains six significant figures.

v. All zeros to the right of a decimal point but to the left of a nonzero digit are **not** significant e.g. 0.000132ml contains three significant figures.

vi. All zeros to the right of a decimal point and to the right of a nonzero digit are significant e.g. 0.08040N and 50.00N each contain four significant figures.

Calculations involving significant figures

You should always remember that mathematical calculations cannot increase either the precision or the number of significant figures of the measured quantities. When we are doing calculations involving significant figures, we apply the following rules:

a. In **addition** and **subtraction**, the number of decimal places in the result should be equal to the smallest number of decimal places of any term in the sum or difference of two terms.

b. In **multiplication** or **division**, the number of significant figures in the result should be equal to the number of significant figures of the list least precise term in multiplication or division.

Example 5

Evaluate the following:

i. 3.76m+46.855m+0.2m

= 50.815m

Applying the first rule, the position of a decimal place in the result should be the same as in 0.2.

Answer: 50.8

ii. 11.3m x 6.8m

= 76.84m^2

Applying the second rule, the number of significant figures in the result will be the same as in 6.8m

Answer: $77m^2$

Activity 4

After we have discussed on how you can precisely observe and record measurements in physics, you must observe and record measurements in experiment 1 on page 2 of Physics practical module in unit 1. Follow all the procedures as guided by the physics practical module and answer all the questions on this practical.

1. 6 Error

An error (inaccuracy) in measurements of any sort is a departure of the output of any measuring system from the value truly representing the quantity being measured. This departure occurs in practice, no matter how sophisticated the measuring equipment is. We usually define error as the uncertainty in measurement.

1.6.1 Types of errors

1. **Systematic errors**: These are errors that appear in measurement due to known causes. They include all inaccuracies that tend to be more in one direction than in the opposite one. We can reduce systematic errors by repeating measurements a number of times. Some important causes of such errors are:
    - Incorrect design or calibration of the instrument
    - Limitation of the method used for the measurement
    - Incorrect reading or interpretation of the instrument
    - Lack of accuracy of the formula being used

2. **Random errors**: These are errors which appear in a measurement due to unknown causes. Random errors have no set pattern and their magnitude depends on the researcher's knowledge of the apparatus used. Taking repeated readings of a quantity and averaging reduces random errors, but not systematic errors.

3. **Gross errors**: these are errors due to sheer carelessness of the researcher. They mostly involve human errors. Gross errors cannot be corrected but we may reduce them. In most cases we generate gross errors by:
    - Recording the observations wrongly
    - Reading the instrument incorrectly
    - Entering wring values of observations in calculations
    - Not caring for sources of error and precision.

1.6.2 Error Analysis

When dealing with errors in physics we oftenly use the following words; true value, absolute error, mean absolute error, relative error and percentage error.

A true value is the arithmetic mean of a large number of readings of a quantity. If during an experiment you have taken a_1, a_2, a_3 ….. a_n as part of n different readings of a physical quantity in an experiment, then the true value of that quantity is

$$\bar{a} = \frac{a_1 + a_2 + a_3 \ldots\ldots\ldots + a_n}{n}$$

$$\bar{a} = \frac{a}{n}\sum_{i=1} a_i$$

We note from the above equation that the true value $\bar{a}$ is the same as arithmetic mean.

We define absolute error as the difference in the magnitudes of true value and the measured value of a physical quantity. We sometimes refer absolute error to actual error. If you observed a_1, a_2, a_3 ….. a_n as you n different readings of a physical quantity in an experiment then the absolute errors are given as;

$$\Delta a_1 = \bar{a} - a_1 ; \quad \Delta a_2 = \bar{a} - a_2 \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad \Delta a_n = \bar{a} - a_n$$

Mean absolute error is the arithmetic mean of all absolute errors in the measured values.

$$\Delta\bar{a} = \frac{|\Delta a_1| + |\Delta a_2| + |\Delta a_3| + \ldots |\Delta a_n|}{n}$$

You should note that $\overline{\Delta a}$ is also called final absolute error. If we have taken the mean value of a physical quantity as $\bar{a}$, $\overline{\Delta a}$ will be the mean absolute error and the measurement of the quantity is finally given as;

$$a = \bar{a} \pm \overline{\Delta a}$$

this means our desired measurement lies between $\bar{a} - \overline{\Delta a}$ and $\bar{a} + \overline{\Delta a}$.

Sometimes you may wish express an error interms of the ratio of absolute error to the mean value (true value) of the quantity, this is known as relative or fractional error.

$$\text{Relative error} = \frac{\overline{\Delta a}}{\bar{a}}$$

When you multiply relative error by 100, the result you get is percentage error.

$$\text{Percentage error} = \frac{\overline{\Delta a}}{\bar{a}} \times 100$$

Example 6

The period of oscillation of a pendulum in an experiment in an experiment is recorded as 2.63s, 2.56s, 2.42s, 2.71s and 2.80s respectively. Find

a. The true mean (true value) period
b. Absolute error in each observation
c. Mean absolute error
d. Percentage error

Solution:

a. We will calculate the mean using equation 1

$$\bar{T} = \frac{2.63 + 2.56 + 2.42 + 2.71 + 2.80}{5} = 2.62s$$

b. We are going to use equation 3 to calculate absolute error.

2.62 – 2.63 = - 0.01s, 2.62 – 2.56 = 0.06s, 2.62 – 2.42 = 0.20, 2.62-2.71= - 0.09s, 2.62 – 2.80 = - 0.18s

c. Applying equation 4 we will get mean absolute error in each observation

$$\overline{\Delta T} = \frac{\sum |\Delta T|}{5} = \frac{|0.01| + |0.06| + |0.2| + |-0.09| + |-0.18|}{5}$$

$$= \frac{0.01 + 0.06 + 0.2 + 0.09 + 0.18}{5} = 0.11s$$

e. Finally, percentage error will be given from equation 6

% Error. $= \frac{\overline{\Delta T}}{T} \times 100 = \frac{0.11}{2.62} \times 100 = 4.2\%$

if we were asked to present the result from this experiment, we would use equation 4.

$T = \bar{T} \pm \overline{\Delta T}$,   Thus T = 2.62±0.11s

Propagation of errors

When we are conducting experiments in Physics we encounter a series of measurements with their associated errors. In order for us to arrive at a final result we have to carry out some mathematical operations like addition, subtraction, multiplication and division.

Error in Addition and Subtraction

If we have measurements of two physical quantities together with their associated errors, a ± Δa and b ± Δb, the sum of these two will be presented as

Q ± ΔQ = (a ± Δa) + (b ± Δb)

This means Q = (a + b) and ΔQ = (Δa + Δb)

Where Q represents a physical quantity and ΔQ represents its associated error

Likewise when subtracting quantities with their associated errors we follow the same pattern as in addition.

Q ± ΔQ = (a ± Δa) – (b ± Δb)

Thus Q = (a –b) and ΔQ = (Δa +Δb)

We have seen that whether we are adding or subtracting, only the actual quantities are added or subtracted but associated errors are always added.

Example 7

Evaluate

a. (25.4±0.1cm) + (16.5±0.1 cm)
b. (25.4±0.1cm) - (16.5±0.1 cm)

Solution

a. Q = 25.4 cm + 16.5 cm = 41.9cm
   ΔQ = (0.1 +0.1) = 0.2 cm
   Answer: Q = 41.9 ± 0.2 cm.

b. Q = 25.4 – 16.5 = 8.9 cm
   ΔQ = 0.1+0.1 = 0.2
   Answer: Q = 8.9 ± 0.2 cm

Error in Multiplication and Division

If we have the same measurements (a ± Δa) and (b ± Δb), an absolute error in their product will be given as

$$Q \pm \Delta Q = (a \pm \Delta a)\,(b \pm \Delta b)$$

Where Q = a × b and ΔQ = $\frac{\Delta a}{a} + \frac{\Delta b}{b}$

Then Q = ab ± ($\frac{\Delta a}{a} + \frac{\Delta b}{b}$)

Similarly if we have two quantities dividing each other $\frac{(a \pm \Delta a)}{(b \pm \Delta b)}$, the evaluation of the error part will be the same as in multiplication, only the quantities will be dividing. For instance, it we are to evaluate P = $\frac{(a \pm \Delta a)}{(b \pm \Delta b)}$

P = $\frac{a}{b}$, ΔP = $\frac{\Delta a}{a}+\frac{\Delta b}{b}$

Fractional error in P = $\frac{a}{b}\pm\frac{\Delta a}{a}+\frac{\Delta b}{b}$

Example 8

Find the fractional and percentage errors in F, if F = (3.5±0.1kg) x (10±1ms^{-2})

Solution

F = 3.5 x 10, ΔF = $\frac{0.1}{3.5}+\frac{1}{10}$ = 0.03+0.1 = 0.13

Fractional in F = F ± ΔF

= 35±0.13

At this point we can calculate percentage error in two ways, either by multiplying fractional error by 100 or working out the sum of percentage errors from different quantities.

Thus, % error = 0.13 x 100 = 13%

Or % error = error in m + error in a

= 0.03 x 100 + 0.1 x 100 = 13%

Error in power of a quantity

When dealing with quantities involving powers we need to be extra careful. On power errors we need to adopt a certain notation: If Δa is the absolute error in the measurement of the quantity a, then we shall record the measurement as a± Δa. For example if T = $(a+\Delta a)^2$

Then T = a^2 and ΔT = $2\frac{\Delta a}{a}$

T = $a^2 \pm 2\frac{\Delta a}{a}$

Example 9

Evaluate D = $(4.0 \pm 0.4 \text{ cm})^2$

D = 4.0^2 = 16, and ΔD = 2 $\frac{0.4}{4.0}$ = 0.2

We can present this measurement as D±ΔD = 16±0.2 cm^2.

As we have seen that the procedure for calculating fractional error in multiplication is the same as in division, we can come up with a general rule for errors in multiplication, division and powers.

Suppose we have a function Q = $\frac{a^l b^m}{c^n}$

$$Q = \frac{ab}{c}, \text{ and } \Delta Q = l\times\frac{\Delta a}{a}+m\times\frac{\Delta b}{b}+n\times\frac{\Delta c}{c}$$

$$Q \pm \Delta Q = \frac{ab}{c}\pm(l\times\frac{\Delta a}{a}+m\times\frac{\Delta b}{b}+n\frac{\Delta c}{c})$$

Activity 5

1. Evaluate the following
   a. (20.0±0.2)+(10.0±0.1)
   b. (30.0±0.3) – (15.0±0.1) + (10.0±0.1)
   c. (25.0±0.5) ÷ (5.0±0.1)
   d. (410±5) × (100± 2) ÷ (200±3)
2. The side of a cube is given as (7.5 ± 0.1) cm. Find the volume of the cube.
3. The length, breadth and thickness of a block of wood are recorded as l = (15.12±0.01) cm, b = (10.15 ± 0.001) cm; h = (5.29 ± 0.01) cm. Find the percentage error in the volume of the block.

1.7 Practice Activity

Having learnt different types of measuring devices such as venier caliper, micro-screw-gauge etc, construct any of the measuring devices using locally available resources. Using your constructed measuring devices, estimate its list count.

Unit Summary

In this unit we have discussed measurements of physical quantities. We have seen that proper and accurate presentation of measurements in physics depends on ones knowledge on dimensional analysis, measuring techniques and error analysis. Since all measurements taken in experiments involve some errors it is very important to know the level of accuracy in our measurements. This unit forms the basis of experimental science and it can be applied in any discipline where there is a need to conduct experiments and present results.

Unit Test (100 Marks)

Answer all questions.

QUESTION 1

a. Define a physical quantity. (2 marks)
b. What is the difference between basic units and derived units? (4 marks)
c. Write 460 km in feet (4 marks)
d. Derive the SI units of gravitational constant G in the equation . $F = \frac{GM_1M_2}{r^2}$

(10 marks)

QUESTION 2

a. Find the dimensions of L, T and M in the dimensional formula of torque

(10 marks)

b. Prove that pressure and energy density have the same dimensions.

(10 marks)

c. The speed of a particle at time t is given by:

$$v = at + \frac{b}{t+c}$$

Find the dimensions of a, b and c. (10 marks)

QUESTION 3

a. Define the term precision as used in physics (2 marks)

b. Figure 1, shows a voltmeter, study it carefully

FIGURE 1.

i. Give a reading on the voltmeter (2 marks)

ii. Express the voltmeter reading with its associated error directly in the units of measurement involved (3 marks)

c. If the current of a 2.33A is passed through a resistance of 10.485Ω, find the voltage across the resistor to correct significant figures (3 marks)

d. Explain any one difference between random errors and systematic errors

e. How many significant figures are in each of the following (10 marks)

i. 205 km

ii. 6000 km

iii. 0.0340 g

iv. 8.00 g

v. 79,000 s

QUESTION 4.

a. Define an absolute error (3 marks)

b. In four sentences briefly explain any two sources of error encountered in measurement of physical quantities (4 marks)

c. What is the percent error in the measurement 8.9±0.2 cm? (6 marks)

d. The force acting on an object of mass m, traveling at velocity v in a circle of radius r is given by;

$$F = \frac{mv^2}{r}$$

The measurements are recorded as m = 3.5 kg ± 0.1kg; v = 20 ms^{-1} ± 1 ms^{-1}; r = 12.5m ± 0.5m. Calculate the maximum possible

i. Fractional error (9 marks)

ii. Percentage error in the measurement of force (8 marks)

Suggested Answers to Unit Activities

Answers to activity 2

1. Basic units do not depend on any other quantity to be formulated where as derived unit depend on basic units to be formulated.
2. a.) 500 TB b.) 0.8 zm
3. a.) 15.7 In b.) 13.41 m/s

Answers to activity 3

1. $[P] = [ML^2T^{-3}]$

2. $[B] = [ML^2T^{-2}A^{-1}]$

Answers to activity 5

1. a.) 30.0±0.3 b.) 25.0±0.5 c.) 5.00±0.05 d.) 205.00±0.05
2. V = 422±17 cm^3
3. 0.36%

UNIT 2: VECTORS

2.0 Introduction

Welcome to Unit 2. As you may recall in unit 1 we covered fundamental quantities and units. In unit 2 we are going to build on the fundamental principals learnt in unit 1. Our movement from one place to another is an example of a vector. We have always known vectors to have both magnitude and direction; however, this is not always the case. Some quantities like current have both magnitude and direction but is not a vector.

2.1 Objectives

By the end of this unit, you should be able to:

- State properties of a vector
- Describe types of vectors with their examples
- Apply graphical and numerical methods in the analysis of vectors
- Resolve vectors into their components

Key terms

Ensure that you understand the key term phrases used in this unit as listed below.

- Vector
- Scalar
- Free vectors
- Sliding vectors
- Fixed vectors
- Equal vectors
- Triangular rule
- Parallelogram rule
- Resolution of a vector
- Component
- Resultant

- Polygon of forces

2.2 Scalar and vector quantities

2.2.1 Scalar and vector quantities

Quantities with magnitude (size) only are referred to as scalar quantities.

Examples of scalar quantities are temperature, distance, volume, speed, mass, density, pressure, etc.

A vector quantity is a quantity with both magnitude and direction. Examples are displacement, velocity, force, acceleration, momentum, etc.

Activity 1

1. State if the following are vectors or not
   a. Water flowing from the top of a hill at constant speed
   b. A boy covering a distance of 20 km
2. Identify any two examples of vectors and scalars in your local area
3. Give an example of a physical quantity which
   a. Has neither unit nor direction
   b. Has a direction but is not a vector
   c. Can either be a scalar or a vector

2.3 Vector Relationships and Representation

2.3.1 Vector Representation

A vector is represented using an arrow and magnitude. An arrow shows direction.

Vector notation: sometimes we use letters with an arrow or a dash on top to denote a vector: $\bar{a}$, $\vec{a}$, $\underline{a}$

2.3.2 Vector Relationships

The following are vector properties or relationships:

1. Vectors are equal if they have the same magnitude, direction and dimension
2. Only vectors of the same units can be added or subtracted.
3. The negative of a vector has the same magnitude but opposite direction
4. Subtraction of a vector is defined by adding a negative vector.
5. Multiplication or division of a vector by a scalar results in a vector for which:
   a. Only magnitude changes if the scalar is positive
   b. The magnitude changes and direction is reversed if the scalar is negative.

Activity 2

1. A child designed a kite to be thrown up in the air. The child flew the kite 400 m due north and then 300 m due south and then 1200 m upwards. What is the net displacement of the kite?
2. What is the minimum number of vectors required to give to give a resultant of zero?

2.4 Types of vectors

2.4.1 Free Vectors

These are vector quantities with both magnitude and direction but without a particular position and a specific point of application in space.

Examples of free vectors are, angular velocity on a rigid body, torque applied to a rigid body, velocity of a particle moving in a straight line.

2.4.2 Sliding Vectors

Vector quantities with both magnitude and direction and have a specific line of action in space but do not have a specific point of application in space. The line of action act at any point along the slide without changing the overall effect.

Examples of sliding vectors include, force applied on a rigid body, the flow velocity of a fluid in a pipe with a uniform section.

2.4.3 Fixed Vectors

Vector quantities with both magnitude and direction and have a specific point of application in space. If the line of action is changed, the overall effect is changed

Examples of fixed vectors are, force on a deformed body, force on a given particle, momentum of a particle, a force applied to the end of a spring.

Activity 3

Having learnt types of vectors, give two examples that you encounter in your daily activities on each of the following:

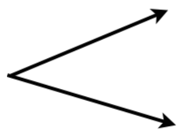
1. Free vectors
2. Sliding vectors
3. Fixed vectors

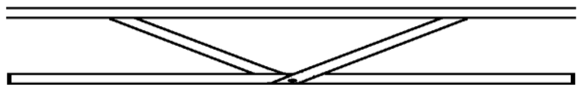
2.5 Force vectors

You may recall that we discussed force as an example of a vector. Inspite of the fact that magnitude and direction, it is very important to specify the direction along the line of action of the action force and the point at which the force is applied. For example, the gravitation force acting on a mass of a body acts towards the centre of the earth and for most purposes this can be taken to be vertically down.

2.5.1 Concurrent forces

These are forces whose lines of action pass through the same point.





2.5.2 Coplanar forces

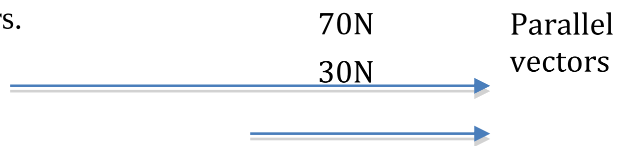
These are forces acting in the plane

2.6 Vector addition and subtraction

Vectors can either act parallel to each other going in the same direction or can act in opposite direction (anti-parallel). Parallel and anti-parallel vectors usually act at 180° to each other.

2.6.1 Parallel and anti-parallel vectors

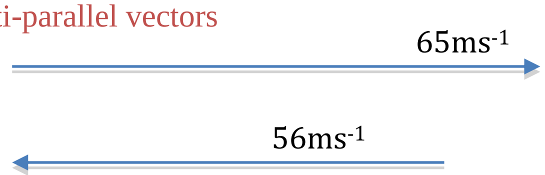
Parallel vectors are simply added where as anti-parallel vectors are subtracted. We normally use ordinary mathematics to calculate the magnitude of parallel and anti-parallel vectors.



Case 1.

Resultant = 70N + 30N = 100N

Case 2. Anti-parallel vectors



Resultant = $65ms^{-1} - 56ms^{-1} = 9ms^{-1}$ in the direction of $65ms^{-1}$

2.6.2 Techniques of adding vectors

When we have vectors acting at any angle other than 180 we use triangular and parallelogram rules to work out their resultant and direction.

TRIANGULAR RULE

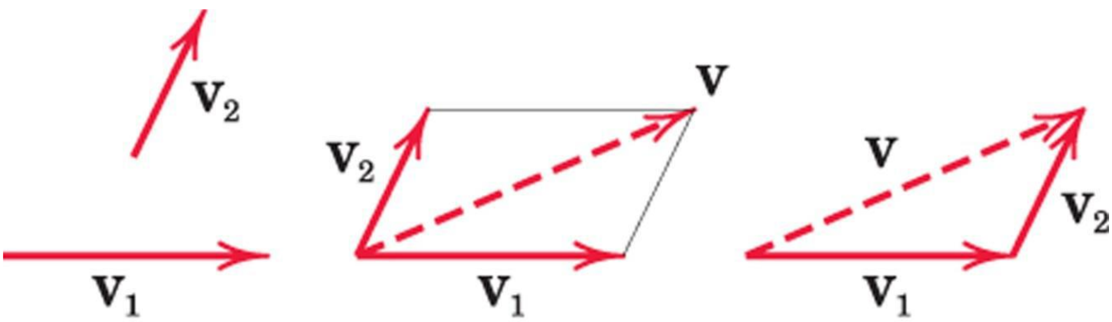
Triangular rule may be applied in two aspects: In numerical method and graphical method. In numerical method we normally apply trigonometry, (Pythagoras theorem, sine and cosine rules). Graphical method is basically scale drawing.

Steps followed when applying triangular rule in graphical method

1. Select an appropriate scale
2. Draw vector V_1 using the selected scale without changing its original direction.
3. Join the tail of vector V_2 to the to the head of vector V_1 without changing the direction of vector V_2
4. Using the same scale draw a third vector joining the tail of V_1 to the head of V_2 and this is referred to as a RESULTANT VECTOR.

Consider the vectors below;

Resultant [R] = V,



PARALLELOGRAM RULE

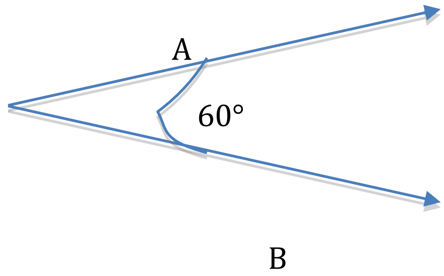
States that, “the resultant of two vectors acting at any point and at any angle is represented by a diagonal formed after completing the vectors into a parallelogram.”

When adding vectors using parallelogram rule, we can either apply the graphical method or numerical method. Most of the steps followed in graphical method are similar to the steps in triangular rule.

Steps followed when applying parallelogram rule

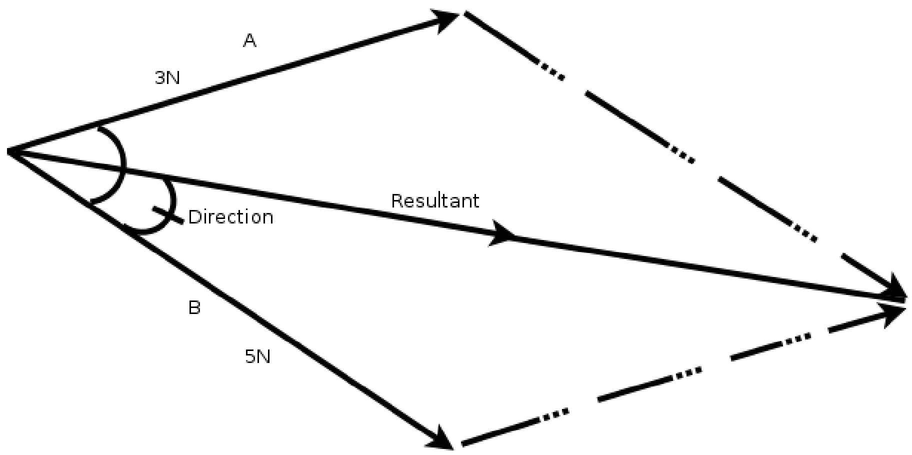
1. Select an appropriate scale
2. Using the selected scale draw vectors A and B joined heat to tail.
3. Complete vectors A and B into a parallelogram
4. Draw a diagonal from where A and B are meeting and that is the RESULTANT VECTOR.

Consider vectors, A and B below



If A is 3N and B is 5N find the magnitude and direction of the two vectors.

Applying graphical method, we have



Measuring the length of the resultant on a graph using an appropriate scale, gives 7.1N as magnitude and 21° from vector B as direction.

Numerical method techniques

We compute both the magnitude and direction of the resultant using trigonometry.

1. Pythagoras Theorem: used when two vectors are acting at right angles

   $R^2 = A^2 + B^2$

2. Cosine Rule

   $C^2 = A^2 + B^2 - 2AB\cos\theta$

3. Sine Rule

   $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$

Example

A force of 3N acts at 60° to a force of 5N. Find their resultant's magnitude and direction.

Solution.

Applying cosine rule

A= 3, B = 5, R=? θ = 120°

$R^2 = 3^2 + 5^2 - 2(3)(5) \times \cos 120°$

$= 34 - 30\cos 120^{\circ}$

$R = \sqrt{49}$

$R = 7N$

Direction of the resultant

In this case we will apply sine rule

$$\frac{\sin 120}{7} = \frac{\sin B}{3}$$

$$\sin B = 3\frac{\text{Sin}120}{7}$$

$$A = 21.3^{\circ}$$

Reading assignment

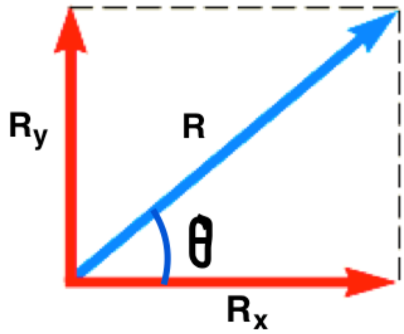
You are asked to read about a triangle of forces and a polygon of forces from other sources. The concepts utilized in these principals are equally significant for this unit.

Activity 4

1. Two students are carrying a pale of water. They each exert 5N and 10N at 30° respectively. Find the magnitude and direction of the resultant force.
2. Two equal forces F have a resultant of 1.5 F. Find the angle between the two forces.

2.7 Resolution of vectors

We usually define resolution of a vector as the process of determining the vertical and horizontal components of a given vector. The vertical and horizontal projections are called components. Lets consider vector $\vec{A}$ below:



Magnitude

$R_x = A\cos\theta$

$R_y = A\sin\theta$

Example:

A man pulls a crate force a force of 60N at 45° to the ground. Resolve this vector into its components.

Solution:

To solve example 3 we simply need to use trigonometric functions A_x and A_y.

$A = 60N$, $\theta = 45°$

$A_x = 60N \cos 45° = 42.4N$

$A_y = 60N \sin 45° = 42.4\ N$

2.7.1 **A system of coplanar forces**

We usually define coplanar forces as forces acting at a single point and in the same plane. If so many vectors are acting from one point you find the resultant vector by resolving each vector into its components and adding all the components together.

Magnitude

$R_x = a_x + b_x + c_x$

$R_y = a_y + b_y + c_y$

$$\therefore\ R = \sqrt{R_x^2 + R_y^2}$$

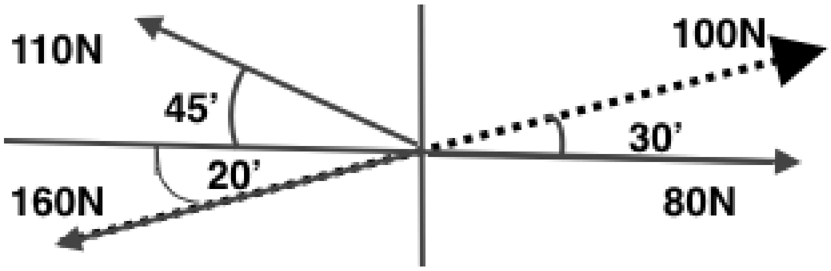
Direction

$$\tan\theta = \frac{R_y}{R_x}$$

$$\theta = \tan^{-1}\left(\frac{R_y}{R_x}\right)$$

Example

Find the magnitude and direction of the resultant of the systems of coplanar forces below:



When resolving vectors into their components we consider the following:

i. All components moving upwards are positive
ii. All components moving downwards are negative
iii. All components moving to the right are positive
iv. All components moving to the left are negative

The best way to solve example 4 is to resolve each particular force into its respective x and y components, for instance, if we take 110N force:

F = 110N, $\theta = 30°$, F_x = negative (to the left), F_y = positive (upwards)

$F_x = - 110 \text{ Cos } 30° = - 95.3°$

$F_y = 110 \text{ Sin } 30° = 55.0°$

If we follow this procedure with each of the forces in example 4 we will get,

FORCE	**X-COMPONENT**	**RESULT**	**Y-COMPONENT**	**RESULT**
100 N	100 Cos 45°	70.7 N	100 Sin 45°	70.7 N
110 N	- 110 Cos 30°	- 95.3 N	110 Sin 30°	55.0 N
160 N	- 160 Cos 20°	- 150.4 N	- 160 Sin 20°	- 54.7 N
80 N	80	80.0 N	0	0
TOTAL	**R_x**	**-95.0N**	**R_y**	**71.0N**

Magnitude

$$R = \sqrt{R_x^2 + R_y^2}$$

$$= \sqrt{(-95^2) + 71^2}$$

$$= 118.6N$$

Direction

$$\theta = \tan^{-1}\left(\frac{R_y}{R_x}\right)$$

$$= \tan^{-1}\left(\frac{71}{-95}\right)$$

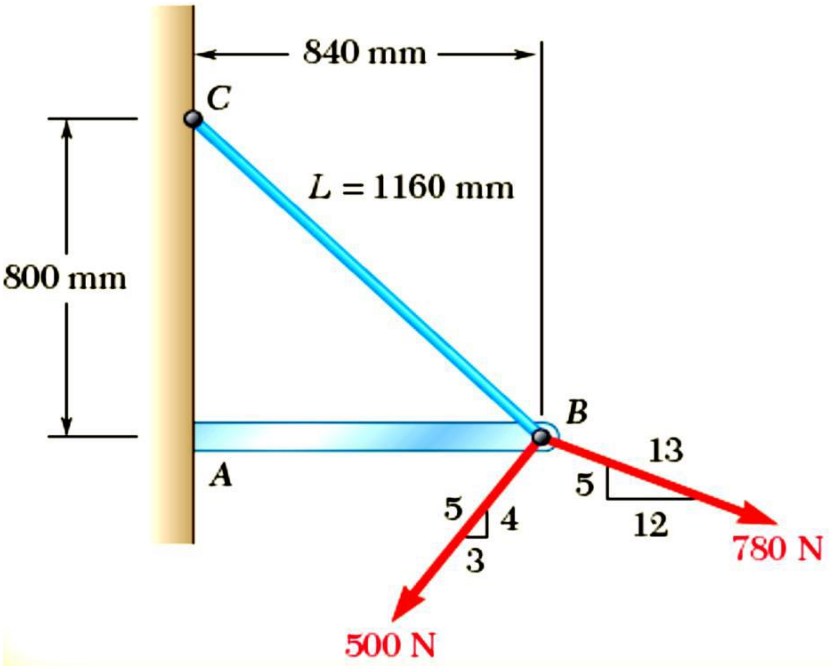
= - 36.8°

The resultant makes an angle of 180° - 36.8° with the positive x-axis, which is 143.2°

In this case our resultant is 118.6N at 143.2°.

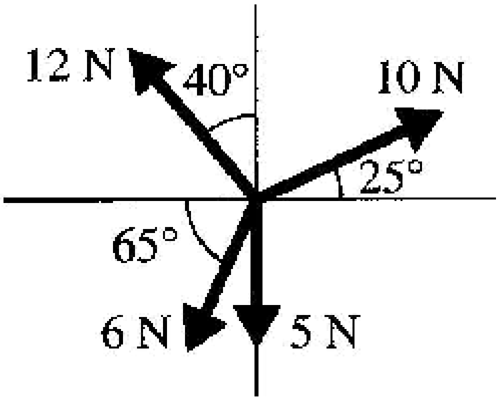
Activity 5

In the figure below determine the resultant of the three forces exerted at point B of bean AB if the tension in BC is 725 N.



2.8 Practice activity

A system of forces at equilibrium is shown below. Find the magnitude and direction of the resultant vector by using polygon rule of vector addition. Hint: select an appropriate scale.

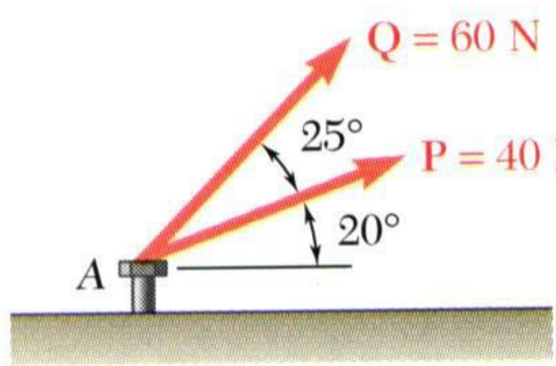


2.9 Unit summary

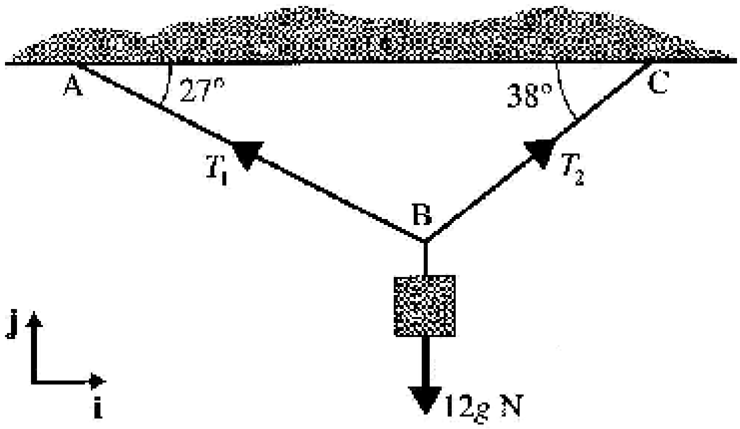
In this unit we have discussed vectors and methods of adding subtracting and resolving vectors. We have seen that vectors can be analyzed using either graphical method or numerical method. Vectors are useful in mechanics because most of the other units built on the concepts of vectors.

Unit Test

1. State three properties of vectors
2. Is a quantity that has both magnitude and direction always a vector?
3. The sum of two vectors at a point is 16N. If their resultant is normal to the smaller force and has a magnitude of 8N, find the forces.
4. State three types of vectors giving an example on each.
5. A boat is sent across a river with a velocity of 8km/h. If the resultant velocity of a boat is 10km/h, what is the velocity of the river?
6. Calculate the resultant of two forces acting on a bolt as shown below



7. Find the value of T_1 and T_2 in the figure below.



Hint: Apply triangle of forces rule or sine and cosine rules.

Suggested Answers to unit Activities

Activity 1

1. Answer
    a. Vector
    b. Not a vector
2. Answer based in your area
3. Answer
    a. Specific weight
    b. Current
    c. Area

Activity 2

1. 1204
2. Two equal forces moving in opposite directions

Activity 3

1. The following are examples of Vectors
    a. Free vectors
        i. Velocity of a car moving in a straight line
        ii. Torque applied to a fixed table
    b. Sliding vectors
        i. The force moving a train
        ii. The force applied to open a sliding gate
    c. Fixed vectors
        i. The force acting on a deformable body like a pillow
        ii. Force acting on a particle of sand

Activity 4

1. 8.66N at 30° with 10N force.
2. 82.82°

Activity 5

Answer: 225.9N at 62.3°

Practice activity

Answer: ~ 3.21 at – 68.3° to the positive direction of x-axis.

Note: your answer may not be as exact as this one, however you are expected to be within a reasonable range.

Unit Test.

1. a). Have dimensions b). Direction and c). Magnitude
2. No, some quantities are not. Electric current has both magnitude and direction but it is not a vector.
3. 6 N and 10 N.
4. You may list any
5. 6 km/h
6. 98 N at 35°
7. T_1 = 102 N and T_2 = 116 N

UNIT 3: BASIC MECHANICAL PRINCIPLES

3.0 Introduction

Welcome to Unit 3!!! In Units 1 and 2, we learnt about fundamental units of physical quantities and vectors, respectively. In Unit 3, we will apply the knowledge and skills acquired from the previous units. If you did not understand the previous two units, please review the material under these units before proceeding to this unit.

In this unit, you will learn about the basic principles applied in design and construction of physical structures and equipment. The design and construction of structures and machines such as irrigation canals, pipelines, farm buildings, roads and tractors employ the principles of mechanics. Mechanics is a branch of Physics which deals with understanding and application of principles associated with forces acting on objects or substances that are either in motion or stationery (at rest). This Unit will introduce you to the basic mechanical principles which you will be required to understand and be able to relate or apply them to solving problems related to design, analysis and installation of physical structures and equipment. We will cover the following in this unit: concepts of force, pressure and moment; beams and beam loading; and material elasticity, stress and strain and friction.

3.1 Objectives

By the end of this unit, you should be able to:

1) Relate the concepts of force, pressure and moment to analysis of forces associated with objects at rest and those in motion and to design of structures
2) Acquire knowledge on different kinds of beam loading and relate them to design of structures
3) Apply the laws governing forces and motion in solving problems related to objects in motion and at rest

Key Terms

Ensure that you understand the key terms used in this unit as listed below.

- Elasticity
- Friction
- Force
- Gravity
- Moment
- Motion
- Pressure
- Strain
- Stress
- Torque

3.2 Concepts of force, pressure and moments

In this section we will discuss three fundamental concepts of mechanics namely force, pressure and moment. You are expected to understand these concepts and be able to relate them to the sections that follow.

3.0.1 Force

We will start by discussing the concept of force. During our day-to-day activities, we use or encounter some forms of forces. For example, when we are walking, sitting, eating, running, etc, some forces are used or experienced. A force can be defined as anything which changes the state of rest or uniform motion of a body. Take note that a force can be in form of a push or a pull and its effect detects its presence. A force can be exerted to:

1) Lift something;

2) Start or stop motion of a body;

3) Deflect (change the direction of) a body;

4) Move a body faster or slower;

5) Bend or break an object;

6) Compress or stretch an object.

You may recall that in Units 1 and 2 we discussed vector and scalar quantities. Force is an example of vector quantities because it has both magnitude (scalar quantity) and direction. Thus, every force acts in a particular direction with a given magnitude. The most common and very important force encountered in our daily living is **gravitational force**. This is the force exerted by the earth on objects near or on the earth surface. Its magnitude is a product of the mass (m) of the object and acceleration due to gravity (g), i.e. m x g or simply, mg. Gravitational force acts towards the centre of the earth. The weight of an object is defined by this product (mg). Its dimension (i.e. unit of measure), as we saw in Unit 1, is a Newton (N).

3.0.2 Pressure

Having talked about force, let us now discuss pressure, a concept related to force. When a force is applied to or by an object, it acts on a specified area and whatever effect the force brings upon the subject depends on how much force is applied per unit area. The force exerted per unit area is referred to as pressure.

Hence, pressure = force/area.

i.e., $P = F/A$

The unit for pressure is a Newton (N) per unit area (i.e. N/m^2, N/cm^2, kN/m^2, etc). One N/m^2 is known as a Pascal (abbreviated Pa). Based on the definition of pressure, it can be seen that if a force of given magnitude is applied to a given area, increasing the magnitude of the force while keeping the area constant will increase the pressure experienced by the subject. Similarly, the pressure will increase if we maintain the magnitude of the force but decrease the area on which the force is acting.

Activity 1: Reflection

Consider yourself walking on muddy ground with a flat shoe and later on wear a sharp pointed shoe and walk on the same ground where you walked with the flat shoe. Which shoe is likely going to make you get stuck in the mud and why?

The answer to this question can be found in the explanation given in the preceding paragraph.

Activity 2: Individual based practical

In this activity, we will analyze the distribution of pressure in a bottle full of water. The materials required include clean water, one plastic bottle of any size (e.g. 250 ml, 300 ml, 500 ml, 1000 ml) a ruler and a sharp object (e.g. a knife) for drilling holes on the bottle.

1) Fill the bottle with water and close it with a suitable lid.
2) Place the bottle on the edge of a table or any elevated surface such as a rock or an inverted bucket.
3) Drill a small hole on the bottle at a location approximately 5 cm from the bottom of the bottle. Observe and write down what happens after drilling the hole.
4) Remove the bottle lid and measure the horizontal distance between the bottle and the farthest point reached by the water coming out of the bottle. Write down your result and state what made the difference in water flow out of the closed bottle and the open one.
5) Quickly fill the bottle with more water and immediately close it tight. Observe and write what happens on the hole that you drilled.
6) Drill another hole at a distance approximately 5 cm from the top of the bottle on the same side as the other hole. Write down what you have observed immediately after closing the bottle
7) Remove the bottle lid and measure the distances of the farthest points reached by the water coming out of the two holes. Explain what happened and why there were any differences in the distances covered by the two outflows.

Example

A person of mass 59 kg exerts all the force on the heel of one shoe on an area of 50 mm^2. Find the following:

a) The force on the surface on which the person stands.

b) The pressure on the surface on which the person stands.

Solution

a) Force on the surface = mg = 59 kg x 9.81 m/s^2 = 579 N

b) Pressure on the surface = Force/Area = 579 N/ 50mm^2

= 11.6 N/mm^2 = 11.6 MN/m^2

= 11.6 MPa

3.0.3 Moment of a force

When a force is applied to a body, it may cause the body to move either in a straight line, to rotate or both. The turning effect about a point is called the **moment of the force**. The impact of a moment of a force depends on a) the magnitude of the force; and b) the perpendicular distance from a point to the line of action of the force. Hence the moment of a force can be defined as the product of the force and the force's perpendicular distance from the axis. In order for us to understand the concept of a moment of a force, let us consider a force F acting at point A on a metal bar or wooden pole AO that is fixed (i.e. whose axis is) at point O as illustrated below.

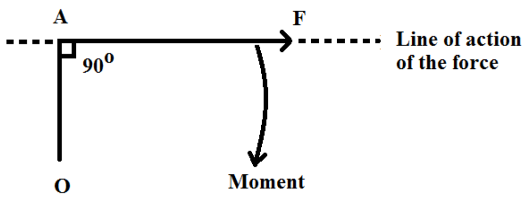


Figure 1. Moment of a force

In Figure 2, the line of action of the force is represented by the dotted line. The perpendicular distance between the line of action and the axis (at O) is OA or AO. Hence, the moment (M_F) of the force (F) = Force (F) x the perpendicular distance from the axis (AO) = F x AO (or M_F = F x AO). Since the units of measure for force and distance are a Newton (N) and metre (m), respectively, then the unit of measure for a moment is a Nm (i.e. N x m).

The moment of the force in Figure 2 is clockwise. Usually, clockwise moments are considered positive while anti-clockwise ones, negative.

3.0.4 A couple of forces and its moment

When discussing a couple, we deal with two forces acting on a body: a) in the same plane (co-planar forces); b) equal in magnitude, c) parallel; and d) acting in opposite directions. The two equal and opposite forces that form a couple have lines of action that do not meet. The two forces always have a turning effect (or moment) called a **torque.** A torque is defined as the product of one force and the perpendicular distance between the forces, i.e. Torque = One force x Perpendicular distance between the forces.

In order to further our understanding of the definition of a torque, let us prove the preceding statement using Figures 3 and 4.

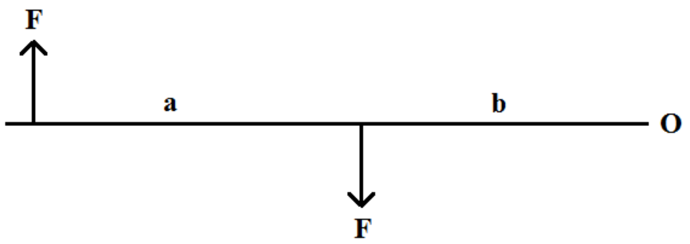


Figure 2

In Figure 3, the force on the far left end is acting in the clockwise direction (+) relative to axis O while the other force is anti-clockwise or counter clockwise (-). The total moment is the summation of the clockwise and anti-clockwise moments.

Therefore, total moment about O = (a + b)F – bF

= aF + bF – bF

= aF

(i.e. One force (F) x the perpendicular distance (a) between the forces)

Similarly, in Figure 4:

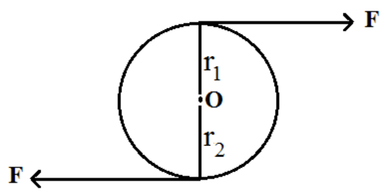


Figure 3

The moments about O are F x r_1 and F x r_2. Hence total moment = $Fr_1 + Fr_2 = F(r_1 + r_2)$. But $r_1 = r_2 = r$, hence total moment = F2r = Fd (since the diameter, d = 2 x the radius, r).

These illustrations indicate that **the moment of a couple is independent of the point of rotation but is dependent upon the perpendicular distance between the forces**.

3.0.5 Equilibrium of forces

A body is said to be in equilibrium when the total effect of all the forces acting on the body is ZERO. A body in equilibrium will therefore have no tendency to either move in a straight line or rotate. The vector sum of all forces acting on the body is zero. This implies that the sum of all forces in the X, Y and Z directions (or planes) is zero. Thus, the summation of the forces is as follows: in the X direction, $\sum F_x=0$; in the Y direction, $\sum F_y = 0$; and Z direction, $\sum F_z = 0$. Overall, $\sum F_{all} = 0$. It follows that the algebraic sum of the moments of all forces about any point is zero, i.e. $\sum M_{all} = 0$. Thus, the sum of clockwise moments = the sum of counter clockwise moments.

3.0.6 Parallel forces

Parallel forces are forces which have the same direction but are not collinear. This means that the lines of action of the forces do not coincide. Such forces may be like or unlike depending on whether they act in the same or opposite sense (direction). When parallel forces are applied to a body free to move, the body is displaced as a whole in some particular direction, and at the same time, rotation of the body occurs.

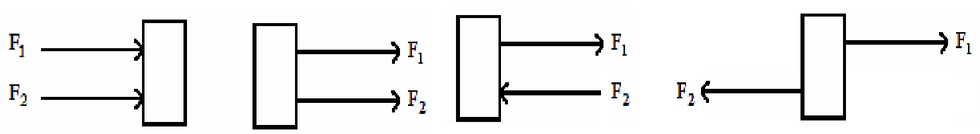


Figure 4. Illustrations of parallel forces

Example

A meter rule is pivoted at the centre and remains balanced. A 50g mass is placed at 30 cm from the pivot. a) How far should a 30g mass be placed in order to keep the rule in equilibrium? b) Find the reaction at the support C.

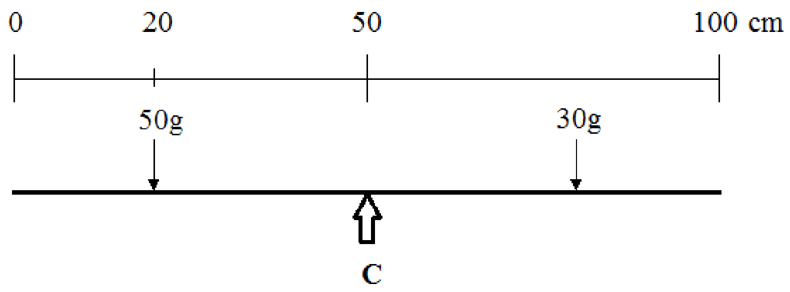


Figure 5

Solution:

a) From our previous discussion, the summation of clockwise moments has to equal summation of anti-clockwise moments under equilibrium conditions. The clockwise moment can be found by multiplying the mass (30 g) by the perpendicular distance from the pivot to where this mass is located. Assuming that the distance is X cm, the moment is 30 g x X. Similarly, the anti-clockwise moment is 50 g x 30 cm.

Hence, clockwise moments = anti-clockwise moments

i.e. 30g x X cm = 50g x 30cm

Therefore, X cm = 50g x 30cm/30g = 50cm

i.e. the 30g mass must be placed 50 cm from the pivot, i.e. at the 100cm mark

b) The reaction at the pivot (support C), is the summation of the vertical components of the loads (masses) because for every action there is an equal and oppositely directed reaction (Newton's law). Since the beam is balanced, summation of forces in the Y direction is zero, i.e., $\sum F_y = 0$. Therefore, reaction at C = 30g + 50g = 80g

3.3 Beams, beam loading and centers of mass and gravity

3.0.7 Beams and beam loading

In the previous example, we looked at forces acting on a metre rule. The metre rule is an example of a beam. A beam is defined as a rigid body subject to transverse loads and reactions. Take note that beams may be classified according to:

i) the way the beam is supported (character of the support); and

ii) the way the beam is loaded (method of loading)

We will start by looking at classification of beams based on the way they are supported. Under this, beams are classified as: a) simply supported; b) cantilever; c) fixed; d) restrained; e) continuous; and f) overhanging. Let us now define each one of them.

a) Simply supported beam

This is a beam that is supported at both ends by either pins or rollers. The pin support resists both vertical and horizontal loads while the roller resists vertical loads only. **Figure 7** is an illustration of a simply supported beam.

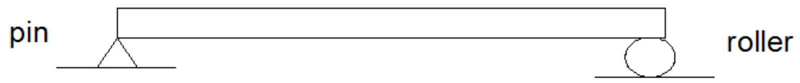


Figure 6. A simply supported beam

b) Cantilever beam

A cantilever beam (**Figure 8**) is fixed at one end and free at the other e.g. a diving board.

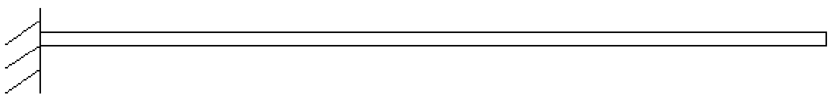


Figure 7. A cantilever beam

c) Fixed beam

This is a beam whereby both ends are fixed (see **Figure 9**)

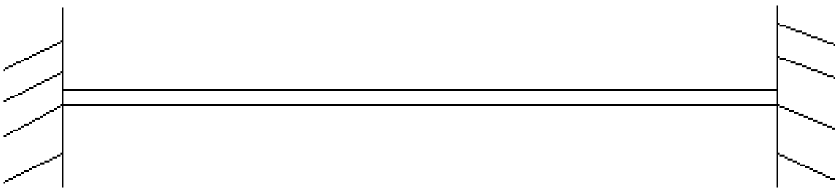


Figure 8. A fixed beam

d) Restrained beam

This is a beam fixed at one end and simply supported at the other as illustrated in **Figure 10**.

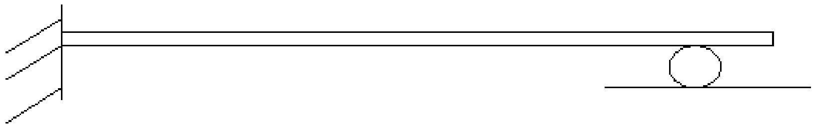


Figure 9. A restrained beam

e) Continuous beam

A continuous beam has intermediate supports hence the term “continuous”. **Figure 11** is an illustration of such.

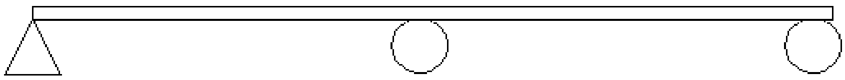


Figure 10. A continuous beam

f) Overhanging beam

Thus is a beam which projects beyond the support. See Figure 12.

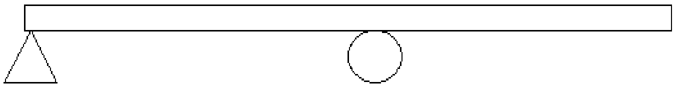


Figure 11. An overhanging beam

Let us now discuss beam classification based on the nature of loading. Beams can either be subjected to a) concentrated or distributed loading.

a) Concentrated loading (or point loading)

Concentrated load is the kind of leading in which the load may be regarded as acting wholly at one point on the beam. **Figure 13** is an illustration of this kind of loading.

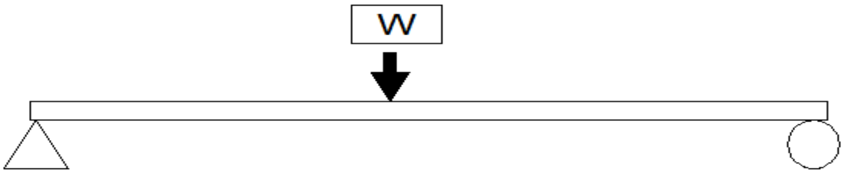


Figure 12. A concentrated load

b) Distributed load

As depicted from the term “distributed”, this kind of load is spread across a length of the beam. The distribution may be uniform or non-uniform as described below.

1) Uniformly distributed load

This is a load which acts all along a given length on the beam. It may act over part (i) or the entire length (ii) of the beam as illustrated in **Figure 14**.

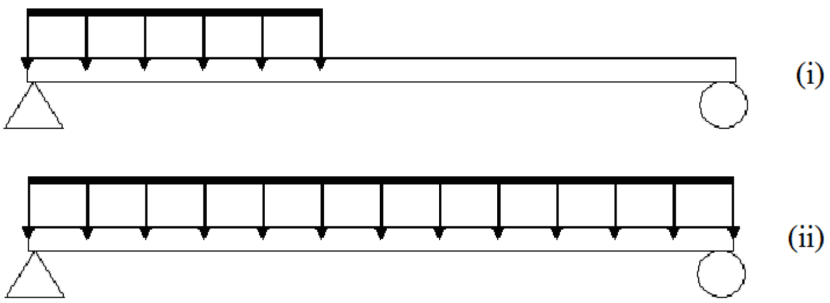


Figure 13. Uniformly distributed loads

2) Non-uniformly distributed load

This is a load which varies from a minimum at one end and a maximum at the other. The load may cover part (i) or the whole length (ii) of the beam (**Figure 15**)

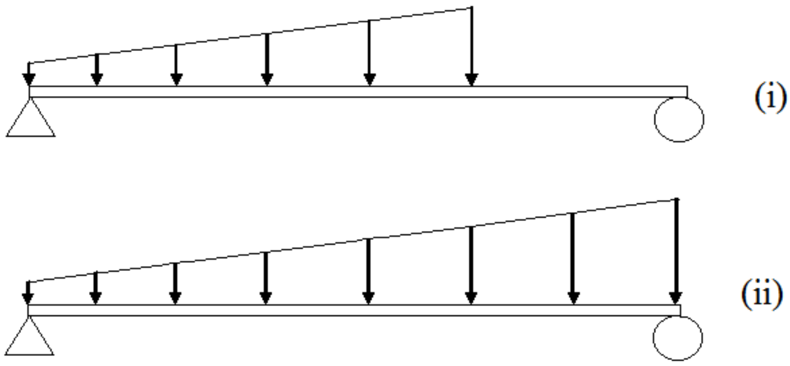


Figure 14. Non-uniformly distributed loads

Activity 5:

Under this activity, we will analyze the effects of different kinds of loading on a beam. The materials required for this exercise are a wooden plank (60 cm long, 5 cm wide, less than or equal to 1 cm thick), two 5-cm diameter stones or two 5-cm diameter poles or any support material that would suspend the wooden block like the way a bridge top is suspended (**Figure 16**).

1. Measure and record the length of the plank.
2. Find five medium sized stones with flat bottoms and placed them evenly along the plank as illustrated below

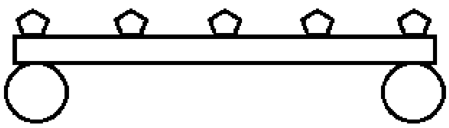


Figure 15. Point loads evenly spaced on the beam

3. Observe and draw an illustration showing what happens to the block (if any)
4. Draw a properly labeled free body diagram of the forces acting on the plank (i.e. arrows representing the directions and magnitudes of the forces)
5. Move all the stones to the mid point of the block (See illustration) and observe what happens to the block

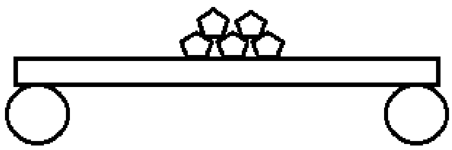


Figure 16. Point loads clustered on the beam

6. Draw a free body diagram of the forces acting on the plank
7. How does this exercise relate to design of bridges?
8. How would you ensure that a bridge that you have designed does not fail or have massive bending when used by different kinds of traffic?

3.0.8 Centre of mass and center of gravity

You may recall that we have discussed different kinds of beam loading. This information is of importance when designing, analyzing and constructing physical structures such as buildings and bridges. Determination of the locations of loads on a given beam is based on the concepts of centre of mass and center of gravity. The centre of mass of an object may be defined as the point at which an applied force produces acceleration but no rotation. On the other hand, the centre of gravity of a body is the point at which the entire weight of the object may be considered to be concentrated. It is the point where the **resultant** force of attraction or of weight of the body acts or appears to act. Strictly speaking, there is a conceptual difference between the centre of gravity and the centre of mass, but for practical purposes they are generally the same point.

3.0.9 Stability and balance

When designing a physical structure, it is important to ensure that the structures are stable and balanced in order to avoid failure. A body in static equilibrium, if left undisturbed, will undergo no translational or rotational acceleration since the sum of all forces and the sum of all the torque acting on it is zero. However, if the object is displaced, three different outcomes are possible:

1. The object may return to its original position, in which case it is said to be in **stable equilibrium.** In this case, the body can not be tipped or overturned unless its centre of gravity is first raised. For any object resting on a support to

be in stable equilibrium, the vertical line drawn through its centre of gravity must fall within its base. In general, the larger the base and the lower the centre of gravity, the more the stable the object.

2. The object will move further from its original position, in which case it is said to be in **unstable equilibrium**. In this case, any slight turning causes the centre of gravity to fall, without first rising.
3. It remains in its new position, in which case it is said to have **neutral equilibrium.** In this case, the centre of gravity is neither raised nor lowered when it is displaced.

3.4 Material Elasticity, Stress and Strain

3.0.10 Basic elastic properties of materials

Let us now look at the basic elastic properties of materials. Any object changes shape are great enough, the object will break or fracture. Knowledge about how much force an object can withstand before it breaks is obviously important in a variety of situations from designing a structure to the action of forces on the human body. If an attempt is made to deform any solid object by changing its size or shape, the object will resist this attempt. Moreover, when the force that has produced a deformation is removed, the object tends to return to its original condition. **All** solids exhibit this property of **Elasticity**. The recovery of an object to its original size and shape will not be complete if the distortion has not been very severe.

3.0.11 Stress and strain

1.4.2.1 Stress

In general, the internal forces acting on extremely small areas of a cut may be of varying magnitudes and directions. These internal forces are vectors in nature and are in equilibrium the external applied forces. When an external force is applied to the member of a structure, a resistance is set up by the material of which the member is composed. The intensity of the forces on the various portions of cut is the resistance to deformation and the capacity of materials to resist forces depends on these intensities. The mobilized internal resistance by a material to any tendency towards deformation is called **stress**. The effect of the force is resisted across the entire cross sectional area of a body.

Therefore, the cross sectional area over which the forces act determines the intensity of the stress.

You will learn that the intensity of the force perpendicular (or normal) to the section is called **normal stress** at the point. Normal stress is designated σ (sigma) and mathematically defined as

$$\sigma = F/A \text{ (force per unit cross sectional area).}$$

Where F= Force acting normal to the cut; and A = corresponding cross-sectional area

Take note that the normal stress which causes tension (stretching) on the surface of cut is referred to as **tensile stress**. On the other hand, the normal stress which is pushing against the cut is known as **compression stress**. These two kinds of stress are illustrated in **Figure 18**.

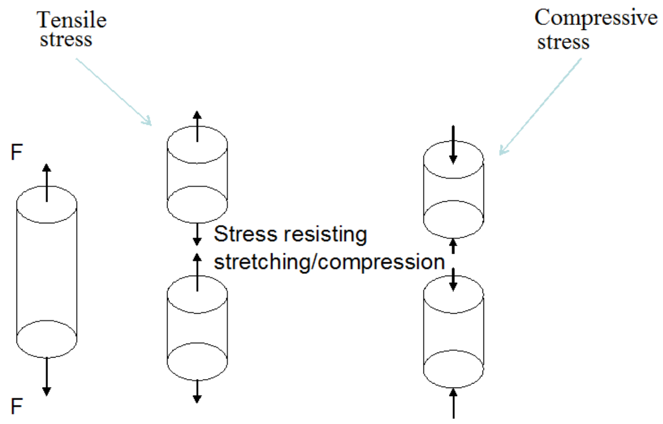


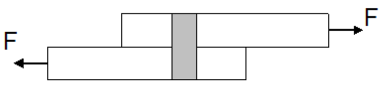
Figure 17. Tensile and compressive stress

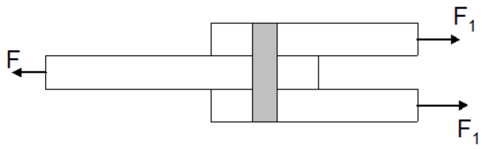
The third kind of stress is s**hear stress**. Shear stress exists between parts of a body in contact when two parts exert equal and opposite forces on each other laterally in a direction tangential to their surface of contact. Shear stress is designated τ (tau), and is mathematically defines as

$$\tau = F/A$$

Where F = Force parallel to the cut; and A = Cross-sectional area

You should note that forces are the products of stresses and the respective areas on which they act. It is the sum of these forces at an imaginary cut that keeps the body in equilibrium.





$\tau = F/A$ $\tau = F/2A$ or $\tau = F_1/A$ (since $F = 2F_1$)

Single shear Double shear

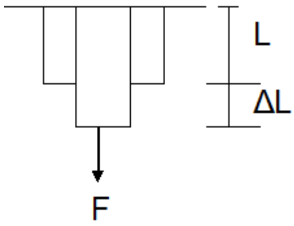
1.4.2.2 Strain

When the material of a body is in a state of stress, deformation takes place thus the size and shape of the body is changed. The manner of deformation will depend on how the body is loaded. Remember that deformation due to an internal state of stress is called **strain**. Any measurement of strain (ε) must be related to the original dimension, e.g. length.

Strain, ε = Deformation/unit of original dimension

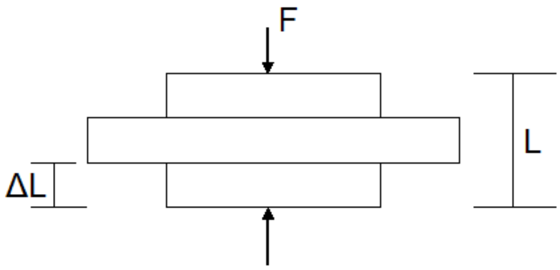
For a body under tension:

Strain, ε = Change in length/original length $= \Delta L/L$



For compression:

$$\varepsilon = \text{Change in length/original length} = \Delta L/L$$



3.0.12 Hooke's Law

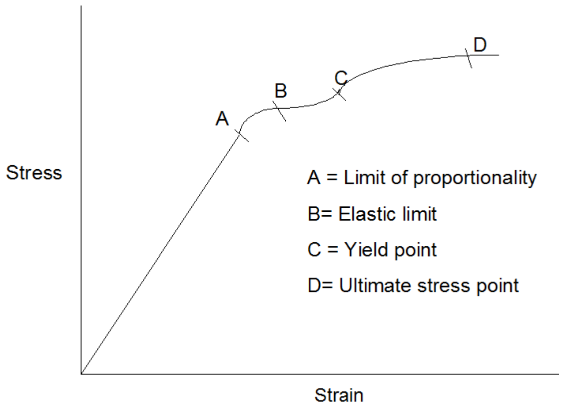
Engineering materials must, of necessity, posses the property of elasticity. This is the property which allows a piece of the material to gain its original size and shape when forces producing a state of strain are removed. This property is defined by Hooke’s law which states that:

“If a bar of elastic material with a uniform cross section is loaded progressively in tension, it is found that, up to a point, the corresponding extension will be proportional to the applied loads”

A more general statement of this law (in terms of stress and strain) is:

“Within the limit of proportionality, the strain is directly proportional to the stress producing it”

During the test, many materials reach a point at which the strain begins to increase very rapidly without a corresponding increase in stress. This is called the **Yield Point**.



Notice that between A and B the diagram is not a perfect line even though the specimen is elastic. Hook's law applies only up to the proportional limit (i.e. up to point A). It is a fact that for a certain distance from the origin, the experimental values of stress versus strain lie essentially on a straight line. For all practical purposes, up to some point A, the relationship between stress and strain may be said to be linear for all materials. Mathematically, this can be expressed by the equation:

$$\sigma = E\varepsilon \text{ or } E = \sigma/\varepsilon = \text{Stress/strain}$$

The equation simply means that stress (σ) is directly proportional to strain (ε) where the constant of proportionality is E. This constant is called **elastic modulus** or **modulus of elasticity** or **Young´s modulus**. As strain is dimensionless, E has the units of stress. Graphically, E is interpreted as the slope of a straight line from the origin to the rather vague point A on the stress-strain diagram.

3.0.13 Factor of safety

It is important for you to realize that when designing physical structures, there is need to slightly over-design them to ensure that they are able to withstand conditions that have not been accounted for in the structural design. This implies increasing the size or strength of the structure beyond your design specifications in order to avoid structural failure under unexpected extreme condition(s). You should note that the additional strength or size to the optimal design specifications is referred to as a "factor of safety". For example, when you are designing a bridge for a given vehicle weight, it is proper to design it to a strength that would withstand slightly more weight than the optimal design specifications require because there may be times when someone drives a slightly heavier vehicle on the bridge. This would result into failure (bending or breakage) of the bridge structure if no factor of safety is added to the optimal design specifications. In practice, most designers use a factor of safety of 10%. Hence, in our bridge design example, if the bridge's optimal design vehicle weight is 3 tons, then the final design specification (including a factor of safety of 10%) would be 3 tons x 110/100 = 3.3 tons.

3.5 Friction

Let us consider an object moving in a particular direction while in contact with another object. The moving object will experience some resistance to motion due to a force that acts in the opposite direction. This force is referred to as frictional force. Frictional forces are caused by adhesion and interlocking of the irregularities of the rubbing surfaces. Friction has both positive and negative effects on the moving object. One of the positive effects of friction is that it makes it possible for objects to be stationary or to stop. For example, let us consider a moving car. For the car to stop, the driver must apply some breaks or the road surface must counteract the motion to such an extent that the car finally stops. In both cases, friction plays role in stopping the car.

The negative effects include the following:

3.0.13.1 It increases the work necessary to operate a machine

3.0.13.2 It causes wear

3.0.13.3 It generates heat which oftentimes does additional damage

We can minimize friction by the following in order to reduce the energy wastage and damage that it can cause:

1) Wheels
2) Bearings
3) Rollers
4) Lubricants
5) Streamlining

We should note that friction also has plays a positive role. The following are examples of situations under which friction is important:

1) Nails and screws are held together by means of friction
2) Power may be transmitted from a motor to a machine by means of a clutch or friction belt
3) Walking
4) Driving a car
5) Striking a match
6) Sewing fabric together

We can group frictional forces into two categories namely, static and kinetic. Static friction is the friction that acts on a body that is at rest while kinetic friction is the friction encountered by a body that is in motion. The relationship between frictional force and the force exerted by a moving or an object that is about to move (i.e. the object's weight) is defined by the coefficient of friction and coefficient of limiting friction, respectively. We will now look at the two coefficients.

3.5.1 Static and Sliding Friction

We can investigate friction forces by pulling a wooden block on a table using a pulley system as illustrated in the Figure below:

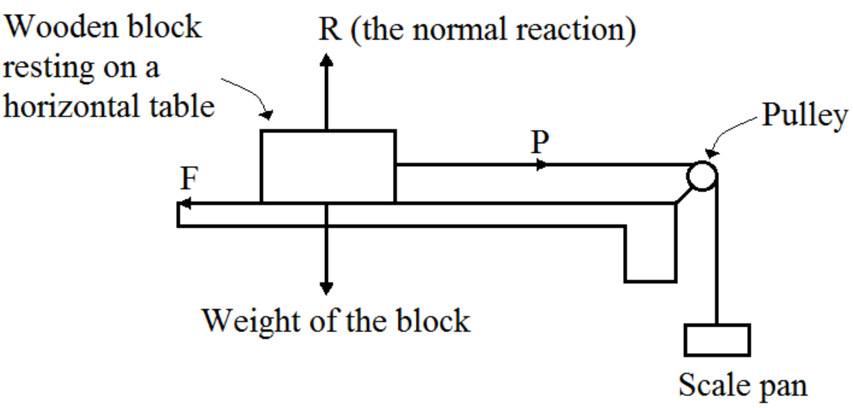


Figure 18. Investigation of friction forces

3.5.2 Coefficient of Friction

Coefficient of friction is the ratio of the frictional force to the perpendicular force pressing surfaces when an object is in uniform motion over another object. We can mathematically define coefficient of friction as

$\mu = F/R$

Thus , $F = \mu R$

where μ is the coefficient of friction; F the frictional force; and R the perpendicular (normal) force.

When a body is horizontal, the normal reaction (force) is equal to the weight of the body. **Figure 20** is an illustration of the different reactions to forces applied to objects placed on smooth surfaces and those on rough surfaces. The reactions are abbreviated R whiled the forces F.

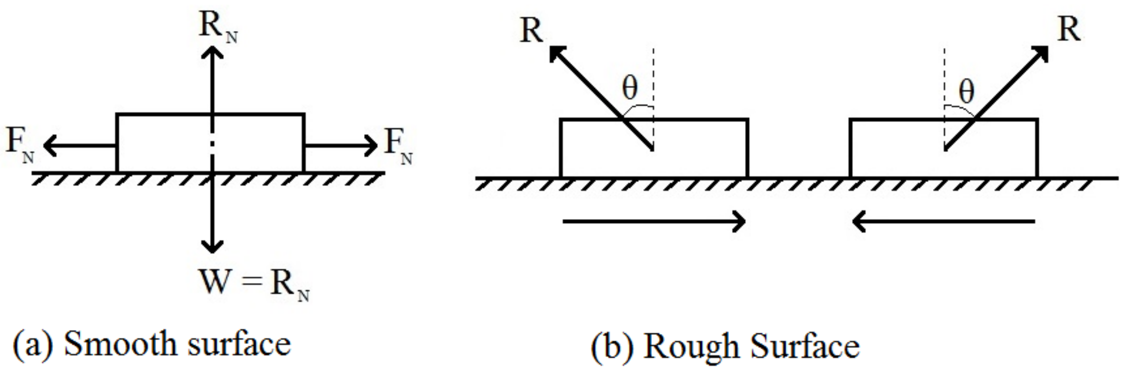


Figure 19. Reaction of force on different surfaces

You will notice from **Figure 20** that the direction of the force pulling an object on a smooth ground is different from the one for an object placed on a rough surface. This is because the rough surface offers more resistance to motion of the object than the smooth surface. In view of this, we will be required to pull the object at an angle relative to the horizontal plane in order for us to be able to move an object placed on the rough surface.

3.5.3 Coefficient of Limiting Friction

Let us now consider an object placed on an inclined plane as given below.

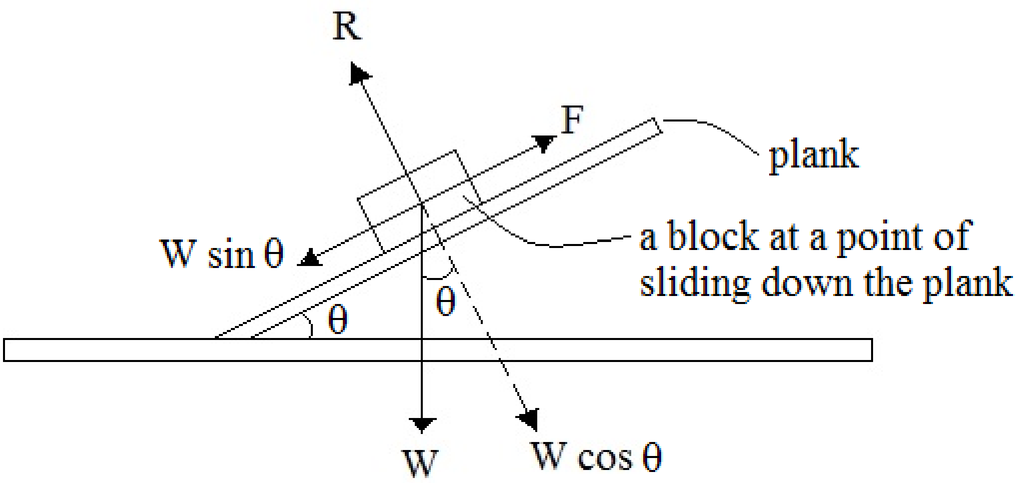


Figure 20. An object on an inclined plane

The object in the figure above is in equilibrium. This implies that R = W cos θ and F = μR = μ W cos θ. If we increase θ slowly, the block will be at the point of sliding down the plane when

W sin θ = μ W cos θ

Thus, $\mu = \frac{W\sin\ \theta}{W\cos\ \ \theta}$

Hence, $\mu = tan\theta$

θ is called the angle of friction, and the maximum angle of inclination for which the static condition exists. When W sin θ exceeds F the block will slide.

The table below presents values of static and kinetic friction for different materials under different conditions.

Table 1. Coefficients of static and kinetic friction for different materials

Material	**μk**	**μs**
Steel on steel, dry	0.40	0.80
Steel on steel, lubricated	0.08	0.15

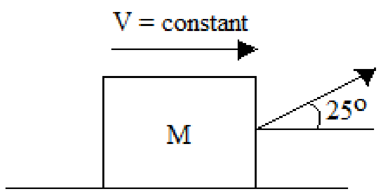
Steel on ice	0.06	0.10
Copper on cast iron	0.30	1.00
Oak on oak (grains//)	0.50	0.60
Rubber on concrete, dry	0.70	1.00
Rubber on concrete, damp	0.50	0.70
Teflon on teflon	0.04	0.04

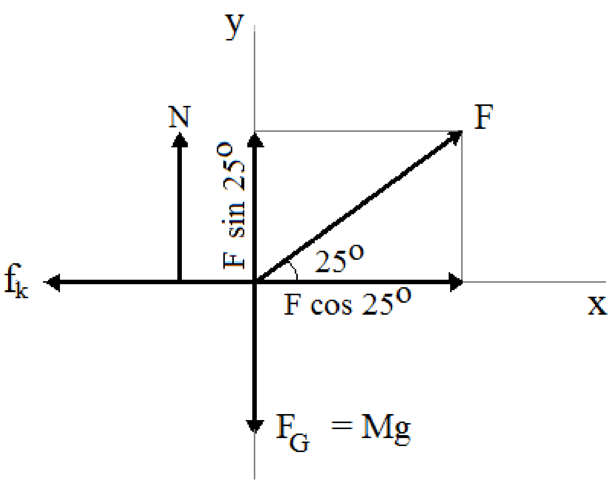
Let us now look at an example.

Example

A block with mass M = 10 kg is pulled along a horizontal surface at a constant velocity by a force F whose magnitude is 50 N. The force makes an angle of 25^o with the horizontal. What is the coefficient of kinetic friction between the block and the plane?

Solution:





$$\sum F_x = 0: \qquad f_k = F\cos 25^\circ$$

$$\sum F_y = 0: \qquad N + F\sin 25^\circ = Mg$$

Making N the subject of the latter formula yields $N = Mg - F\sin 25^\circ$

We can now find the coefficient of kinetic friction by dividing f_k by N:

$$\mu_k = \frac{f_k}{N} = \frac{F\cos 25^\circ}{Mg - F\sin 25^\circ}$$

$$\mu_k = \frac{(50N)(\cos 25^\circ)}{(10kg)\times 9.81m/s^2 - (50N)\times \sin 25^\circ}$$

$$\underline{\mu_k = 0.59}$$

Unit Summary

In this unit, we have covered the basic principles of mechanics. We have discussed concepts of force, pressure, torque, moments of a force, loading of beams, laws associated with forces acting on objects in motion and at rest, and friction. The knowledge and skills acquired in this unit have given us a foundation for analyzing and solving problems related to mechanics.

3.6 Unit Test

3.6.1 State conditions of a couple.

3.6.2 Define Young’s modulus.

3.6.3 Assume Young's modulus for a bone is 1.50×10^{10} N/m^2. The bone breaks if stress greater than 9.8×10^8 N/m^2 is imposed on it.

a) What is the maximum force that can be exerted on the femur bone in the leg if it has a minimum effective diameter of 2.50 cm?

b) If this much force, in (a) above, is applied compressively, by how much does the 25.0 cm long bone shorten?

Suggested Answers to the Unit Test

Here are the suggested answers for the Unit Test:

Model Answers for Activity 1.7.1:

a) A pair of forces acting in the same plane (co-planar forces)

b) The forces must be equal in magnitude

c) The forces must be parallel to each other

d) The forces must be acting in opposite directions

Model Answers for Activity 1.7.2:

Young's modulus is the ratio of stress to strain. Since stress is force per unit area (i.e. F/A) and strain is the amount of deformation per unit of original length (i.e. ΔL/L), then Young's modulus (E) is F/A divided by ΔL/L, i.e. E = F.L/A.ΔL.

Alternatively:

Young's modulus is the slope of the straight line from the origin to the limit of proportionality on the stress-strain graph.

Model Answers for Activity 1.7.3:

a) $F = 4.8 \times 10^5$ N

b) $\Delta L = 16.67 \times 10^{-3}$m

UNIT 4: MOTION

4.0 Introduction

Welcome to Unit 4!!! In Units 2 and 3, we learnt about vector quantities and basic mechanical principles, respectively. In this unit, we will apply the knowledge acquired from the previous units in another important physical concept of motion. If you did not understand the previous three units, it is recommended that you review these units before proceeding to Unit 4. In this unit, you will learn about motion in a straight line and motion in a circular path.

Objectives

4.1.1.1 By the end of this unit, you should be able to understand and apply concepts of motion in a straight line and in a circular path.

4.1.1.2 Relate motion in a circular path to motion in a straight line

4.2 Motion

The science which describes and predicts the conditions of rest or motion of bodies under the action of forces is called MECHANICS

A part of mechanics that describes motion is known as KINEMATICS

DYNAMICS: a part of mechanics that relates motion to forces associated with it and to the properties of the moving object

PARTICLE KINEMATICS: A moving object can rotate (e.g. a satellite) or vibrate (e.g. falling rain droplets)

Complications associated with rotation and vibration can be avoided by considering the motion of an ideal body called a PARTICLE.

A particle is mathematically treated as a point and an object without extent though there is no such thing as an object without extent in nature

Our discussion on motion will dwell on a particle for simplicity's sake

Objects that move without rotation have every part of the object move in the same direction. This is referred to as TRANSLATIONAL MOTION.

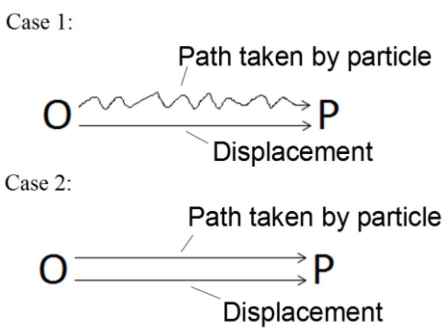
LINEAR MOTION: Motion along a straight line

In this section we will discuss motion of objects along a straight line. The major components of the section are velocity and acceleration. You are expected to understand these and be able to apply them in solving problems related to motion of objects.

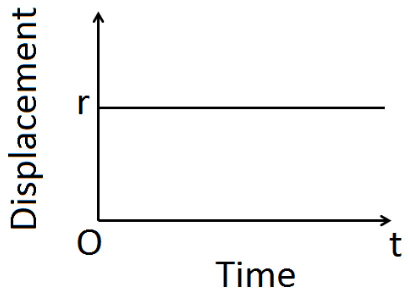
4.3 Speed, Velocity and Acceleration

In the previous Unit on vectors you learnt about scalar quantities and vector quantities. To recap, scalar quantities have magnitude only while vector quantities have both magnitude and direction. Both velocity and acceleration are vector quantities. In order to understand linear velocity and acceleration, it is important to first of all discuss **linear displacement**, a vector quantity related to these two motion properties. Linear displacement of a point is the straight line connecting the initial and final positions of a point in space after the point has moved. Displacement is a vector quantity. It represents the distance (magnitude) covered by a body. The movement is in a specified direction from initial to final position. In summary, displacement comprises distance covered and the direction of motion.

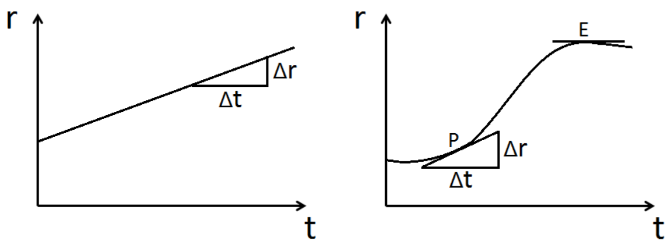
The magnitude of linear displacement is equal to the distance traveled along a path if the path of the latter is a straight line. See illustration below.



Constant Displacement Graph



Non-Constant Displacement Graphs



a) Uniformly increasing displacement

b) Non-uniformly increasing displacement

4.1.2 Speed

Thus, they act in a particular direction at a given magnitude. Their scalar quantity (magnitude or numerical value) is **speed**. Speed is defined as how far (the distance, s) an object moves per unit time, t. The distance (s) traveled over a time interval (t) is referred to as **average speed** (i.e. average speed = s/t). When an object moves in such a way that the distance traveled is the same in successive time intervals, the object is said to be traveling at a **constant speed**.

The speed of an object at an instant of time (e.g. the speed of a car read off a speedometer at a given time) is called its **instantaneous speed**. For instantaneous speed, the interval is so short (very close to zero) that the speed can be considered constant during that time. In calculus notation, instantaneous speed is the derivative of distance (s) with respect to time (t) i.e. ds/dt. In simple terms, this (ds/dt) is the rate of change of distance (s) with respect to time (t). An object can also move with continuous changes in speed. In this case, the object is said to move at a **varying speed**. The instantaneous speed of an object moving at varying speed is found in a similar manner to that of an object moving at constant speed (i.e. ds/dt).

4.1.3 Velocity

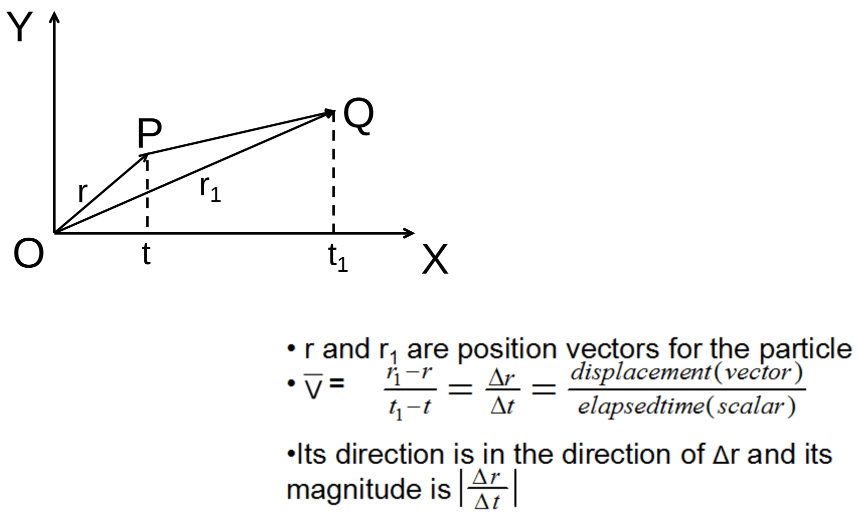
Having discussed speed (the scalar quantity of velocity and acceleration), let us now talk about velocity and acceleration. In simple terms, v**elocity** is a vector quantity of magnitude equal to its speed acting in a given direction. If an object undergoes a vector displacement *s* in time interval *t*, then

v = velocity

= vector displacement/time taken

= *s*/*t*

The direction of the velocity vector is the same as that of the displacement vector. This implies that when an object is displaced in a particular direction (moved a certain distance), the direction of the velocity vector is the same as that of the displacement. We can define average velocity as half of the sum of the initial (u) and final (v) velocities, i.e. Average Velocity, $V_{average}$=½ (u+v).



4.1.4 Instantaneous Velocity

Let us now consider a particle moving at variable velocity. Its instantaneous velocity, v, at an instant time, t, can be obtained from the average velocity by choosing shorter and shorter intervals Δt and Δr. Thus, in calculus notation, instantaneous velocity (v) is a limit of Δr/Δt as Δt approaches zero. This can be written as:

$$v = \lim_{\Delta t \to 0} \frac{\Delta r}{\Delta t} = \frac{dr}{dt}$$

The magnitude, υ, of instantaneous velocity v is the speed and simply the absolute value of v, i.e.,

$$\upsilon = |\mathrm{v}| = \left|\frac{dr}{dt}\right|$$

You may recall that speed is a magnitude of a vector hence it is positive. Velocity of a particle will change only if either magnitude or direction or both change. Let us consider the illustrations in **Figure 22**.

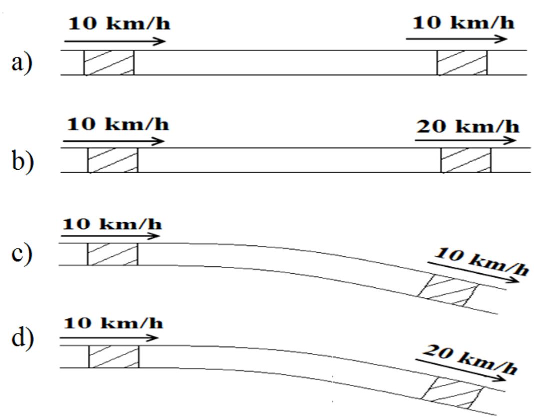


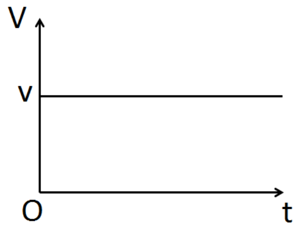
Figure 21. Illustrations of changes in velocity

With reference to **Figure 22**, there is no change in velocity in a) because both the speed and direction have not change. On the contrary, in b) the speed has increased from 10 km/hr to 20 km/hr. In this case, there is a change in velocity despite that the direction remains the same. A velocity change has also occurred in c) because the direction has changed despite the speed remaining the same. Finally a velocity change has also occurred in d) due to changes in both speed (from 10 km/hr to 20 km/hr) and direction.

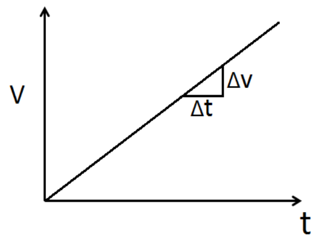
4.1.5 Uniform Velocity

Motion along a straight line at constant speed

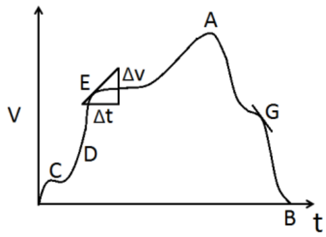
The body has a constant rate of change of distance moved with time in a specified direction



NON-UNIFORM VELOCITY



a) Steady velocity increase



b) Non-steady velocity

Velocity Graphical Representation Summary

Now let us summarize the graphical representations of velocity. The instantaneous velocity of an object at a given time is the slope of the s versus t graph at that time. The instantaneous acceleration of an object at a particular time is the slope of the v versus t graph at that time. For constant velocity motion, the s versus t graph is a straight line and for constant acceleration motion, the v versus t graph is also a straight line.

4.1.6 Relative Velocity

From our knowledge that the earth is always moving (rotating on its axis and revolving around the sun), we will all agree that there is no point on earth that is completely at rest. Also, when we measure the velocities of given moving objects, the measurements are made to the surface of the earth. In this case, the measurements are taken on assumption that the point from which the measurement is made on the earth's surface is fixed (i.e. not in motion due to the earth's movements). In a similar manner, it is proper to regard any moving point A "fixed" and measure the velocity of any other point B relative to point A. Thus, the velocity of B can be obtained as it would appear to the observer moving with point A.

The velocity of B relative to the earth (V_B) is then made up of two parts:

1) V_{BA}, the velocity of B relative to A (as if A was at rest)
2) V_A, the velocity of A relative to the earth (i.e. the velocity of A)

Two approaching trains, each with a speed of 80 km/h with respect to the earth, appear to be approaching at 160 km/h to an observer on each train. Similarly, when a car traveling at 90 km/h is passing another traveling at 75 km/h, the speed of the first car relative to the second determined from the difference between the two speeds as 90 km/h – 75 km/h = 15 km/h.

There are two scenarios that exist when we talk about relative velocities of two objects (object A and object B). The first one is whereby both objects are in a straight line while the other one is when the two are not in a straight line. We will first of all consider objects that are in a straight line and later on look at those that are not.

a) Both A and B in a straight line:

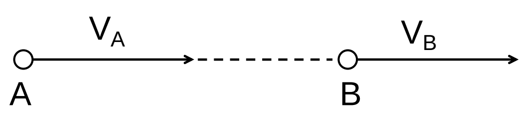


Figure 22. Objects in a straight line

From **Figure 23**, the velocity of A relative to B (abbreviated V_{AB}) is equal to the velocity of B relative to A (abbreviated V_{BA}). Thus, $V_{AB} = V_{BA}$. We can also define the velocity of B relative to A (V_{BA}) as the difference between the velocity of B and velocity of A, bearing in mind that the velocities of A and B respectively are measured relative to a particular point on the earth's surface. Therefore, $V_{BA} = V_B - V_A$.

b) A and B not in a straight line:

Let us consider the same objects A and B not moving along the same straight line as illustrated in **Figure 24**. The velocity of object A is V_A while that of B is V_B. V_A is moving diagonally to the right while B is moving horizontally to the right.

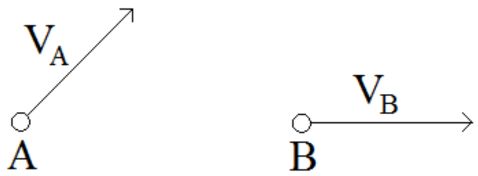


Figure 23. Objects not moving along the same straight line

From this scenario, we can come up with a vector diagram. Remember that in Unit 2 you learned that when representing vectors graphically, there is need to maintain their magnitudes and directions. We will apply that principle here when drawing the vector diagram. We need to bear in mind that we are not interested in finding the resultant force but the relative velocities of the two objects whose origin is the same (O). Hence, we will not join the velocity lines (vectors) head-to-tail but just connect them to the origin while maintaining their directions and magnitudes. In other words, we will draw vector **oa** parallel to V_A and **ob** parallel to V_B. Let us now look at the velocity diagram (**Figure 25**) that results based on these pieces of information.

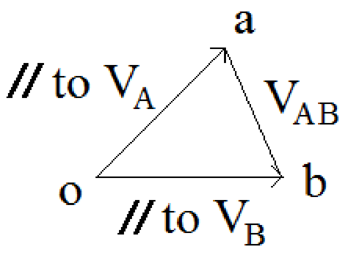


Figure 24. Illustration of relative velocity of objects that are not in a straight line

From **Figure 25**, absolute velocity (velocity relative to the earth) is always measured from point O in the diagram. We can find the velocity of B by vector addition of **oa** and **ab**.

Examples

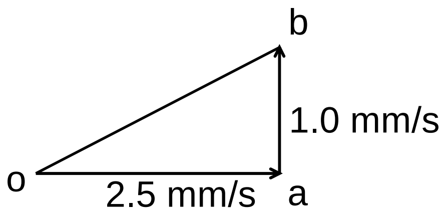
1) A tool is traversed across a lathe bed at 1 mm/s relative to the slide. The slide is traversed at 2.5 mm/s along the lathe. What is the velocity of the tool?

Solution:

i) Draw **oa** horizontally to represent the velocity of the slide, 2.5 mm/s.

ii) From **a** draw **ab** normal to **oa** to represent the velocity of the tool across the bed, relative to the slide, 1 mm/s

iii) Then **ob** represents the velocity of the tool



The speed of the tool (2.7 mm/s) can be found by measuring the length of **ob** on the velocity diagram. The direction of motion for the tool corresponds to the direction of **ob** which makes an angle of 21 degrees 48 minutes (21^o 48') with **oa** (i.e. with the axis of the lathe).

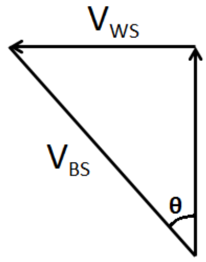
4) A boat can travel 1.20 m/s in still water.

a) If the boat heads directly across a stream whose current is 0.75 m/s what is the velocity (magnitude and direction) of the boat relative to the shore?

b) What will be the position of the boat, relative to its point of origin, after 3.0 s?

Solution:

Here is the velocity diagram for the problem:



As shown, the boat is pulled downstream by the current and its velocity relative to the shore, V_{BS}, is the vector sum of its velocity relative to the water, V_{BW}, and the velocity of the water relative to the shore, V_{WS}

a) To find the velocity of the boat graphically, draw the velocity diagram to scale and measure θ and V_{BS} from the velocity diagram. If done correctly, you should obtain $\theta = 32^o$ and $V_{BS} = 1.4$ m/s.

Alternatively, we can find the velocity of the boat analytically using the Pythagoras Theorem as follows:

V_{BS} is the hypotenuse of the right-angled triangle formed in the velocity diagram, hence

$$V_{BS} = \sqrt{V_{BW}^2 + V_{WS}^2}$$

$$V_{BS} = \sqrt{(1.2)^2 + (0.75)^2}$$

$$V_{BS} = 1.4 m/s$$

b) Let us now find the position of the boat relative to its point of origin after 3.0 seconds. In a) above, we have found that the velocity of the boat (V_{BS}) is 1.4 m/s. Since we know that velocity equals distance over time (s/t), then the distance covered by the boat in 3 seconds can be found by multiplying the velocity of the boat by time as follows:
Distance, s = vt = 1.4 m/s x 3.0 s = **4.2 m**.

Hence the boat will be 4.2 m away from its point of origin in 3 seconds.

4.1.7 Acceleration

The velocity of an object either increases or decreases under certain circumstances. When the velocity increases, the rate of change of velocity per unit time is referred to as **acceleration**. Similarly, the rate of change of a decline in velocity is known as **deceleration**. Deceleration can literally be referred to as **negative acceleration**.

Since acceleration occurs over a time interval (t), we can mathematically define average acceleration (a) as follows:

$$\text{Average acceleration, } \bar{a} = \text{Change in velocity/time}$$
$$\text{i.e., } \bar{a} = (v - u)/t$$

Where u is the initial or starting velocity; v is the final velocity after time t; and

t the time interval over which the change occurred.

Remember that acceleration is a vector quantity. Its direction is in the direction of the change in velocity (v – u).

4.1.7.1 Uniform accelerated motion along a straight line (Linear motion)

When the rate of change in velocity is uniform over time, the object is said to have undergone uniform acceleration. In this case the acceleration vector is constant and along the line of the displacement vector. This scenario can be represented graphically as shown in **Figure 26**.

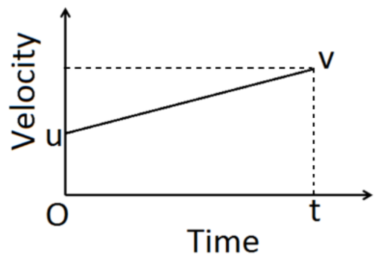


Figure 25. Uniformly accelerated motion graph

If we represent the displacement by s (positive if in the positive direction, and negative if in the negative direction), then the motion can be described using five motion equations for uniformly accelerated motion:

a = (v - u)/t (Eq 1)

From (Eq 1), **v = u + at** (Eq 2)

The distance covered, s = average velocity x time

s = ½ (u + v)t (Eq 3)

Using (Eq 2), substituting (u + at) for v in Eq 3 yields:

s = ½(u + u + at)t

Therefore, $\mathbf{s = ut + \frac{1}{2} at^2}$ (Eq 4)

When we make t the subject of Eq 1, we have t = (v – u)/a. Substituting (v – u)/a for t in Eq 4 yields:

$s = u (v – u)/a + ½ a \{(v –u)/a\}^2$

$= (uv – u^2)/a + ½ a((v^2 -2uv + u^2))/a^2$

$= 2 (uv – v^2) + (v^2 -2uv + u^2)/2a$

Or: $2as = 2uv – 2u^2 + v^2 – 2uv + u^2$

i.e., $2as = v^2 - u^2$

Hence, $\mathbf{v^2 = u^2 + 2as}$ (Eq 5)

Graphical representation summary:

Here is a summary of the graphical representations of velocity and acceleration:

- The instantaneous velocity of an object at a certain time is the slope of the s versus t graph at that time.
- The instantaneous acceleration of an object at a certain time is the slope of the v versus t graph at that time.
- For constant velocity motion, the s versus t graph is a straight line.
- For constant acceleration motion, the v versus t graph is a straight line.

4.1.7.2 Acceleration of Freely Falling Objects

We will now look at another motion scenario involving falling of bodies in space. In the absence of air we find that all bodies regardless of their size, weight, or position, fall with the same acceleration at the same point on the earth's surface, and if the distance covered is not too great, the acceleration remains constant throughout the fall. This ideal motion, in which air resistance and the small change in acceleration with altitude are neglected, is called "FREE FALL".

Acceleration of a freely falling body is called ***the acceleration due to gravity*** and is denoted *g*.

- Near the earth's surface its magnitude is approximately 9.81 m/s and its direction is down towards the centre of the earth.
- The value of this gravitational acceleration varies from place to place but the variation is so small (<1%) that it is neglected.

The equations that we discussed earlier in this unit (Eq 1 through 5) for constant acceleration are applicable for problems in free fall.

Examples

1. A cyclist starting from rest attains a speed of 20km/h in 50 seconds while traveling in a straight line. Determine his average acceleration?

Solution:

Using equation 1 (Eq 1)

$a = (v – u)/t$

$= (20km/h – 0)/50s$

$= 20km/h \times 1/50s \times (h/3600s) \times (1000m/km) =$ **$0.11m/s^2$**

2. A body, starting with a velocity of 10km/h moves with a constant acceleration of $2.5m/s^2$. How far and how long must it travel before it attains a speed of 45km/h?

Solution:

Using equation 5 (Eq 5)

$s = (v^2 – u^2)/2a$

$= (452\ km^2/h^2 – 10^2\ km^2/h^2)/2 \times 2.5\ m/s^2$

= **29.71m**

Using equation 2 (Eq 2)

$v = u + at$

Hence, $t = (v - u)/a$

$t = (45km/h – 10km/h)/\ 2.5m/s^2$

= **3.89 s**

Note that it is better to avoid using quantities that one has calculated to find another because if that value is wrong then so too will the one found using it.

4.1.8 Newton's Laws of Motion

Having talked about velocity, let us discuss some laws governing motion of objects. It is important for us to understand these laws in order to be able to apply them when solving problems related to motion. The laws were developed by Sir Isaac Newton and are referred to as Newton's laws of motion. We will now look at the laws. You will be expected to know these laws by heart and be able to apply them in solving problems related to motion of objects.

1) Newton's First Law of Motion states that:

"Every body continues in its state of rest or uniform motion in a straight line unless acted upon by some external force".

The tendency of a body to maintain its state of rest or of uniform motion in a straight line is called **inertia**. The inertia of a body is determined by its mass. The more inertia a body has, the harder it is to change its state of motion. In other words, it is harder to make such an object to start moving (from rest) or to stop it when it is moving. It is also harder to change its motion sideways out of a straight path.

2) Newton's Second Law of Motion

Having looked at Newton's first law of motion, let us now discuss the second law of motion which states that:

"The rate of change of momentum (where momentum = mass x velocity) of a body is directly proportional to the resultant external force which is producing the change".

We can mathematically define this law using the following equation:

$$F = \frac{d(mv)}{dt}$$

where F = applied force

$\frac{d\,(mv)}{dt}$ = the rate of change of momentum

We will have a more detailed discussion of momentum in a latter section after the presentation on the third law of motion. In the meantime, let us look at an alternative way of stating the second law of motion which reads:

"The acceleration of an object is directly proportional to the net force acting on it and is inversely proportional to its mass".

Again, we can mathematically write this statement as

$$a \propto F/m$$

i.e. F = ma

The above equation can only be applied where the mass is constant. It is also worth noting that the direction of the acceleration is in the direction of the applied net force.

We are now left with only one law of motion known as Newton's Third Law of motion as presented below.

3) Newton's Third Law of Motion states that:

"If a body A exerts a force on a body B, then B exerts an equal and oppositely directed force on A"

We can alternatively state this law as:

"To every action, there is an equal and opposite reaction"

This law explains why you are able to sit on a chair without experiencing any chair movement or breakage when you are on it. It implies that the chair applies an equal and oppositely directed (upward) force on you in order for your body weight (that is acting downwards on the chair) to be supported by it, otherwise, the chair will fail (break) or get flattened between you and the ground if it is not providing an equal and opposite reaction.

4.1.9 Momentum

Let us now have a more detailed discussion of momentum by assuming that the velocity of an object changes from V_1 to V_2 in time t. In other words, the change in velocity from v_1 to v_2 occurred during a time interval t (t is also referred to as elapsed time). Now, as we saw in the previous section when we defined Newton's Second Law of Motion, momentum (M) is the product of mass (m) and velocity (v). Hence, the change in velocity from v_1 to v_2 results in change in momentum from mv_1 to mv_2 (i.e., change in momentum = $(mv_2 – mv_1)$). By definition (from motion Eqn number 1), acceleration, $a = (v_2 - v_1)/t$. Since the alternative expression of Newton's Second Law of Motion states that F = ma, then substituting the expression "(v2 - v1)" for "a" in this equation (F = ma), yields $F = m(v_2 – v_1)/t$. We may as well write this equation as $F = (mv_2 - mv_1)/t$.

Now, let us make "($mv_2 - mv_1$)" the subject of the formula. This will give us: ($mv_2 - mv_1$) = Ft. As we saw in the paragraph above, the expression "($mv_2 - mv_1$)" is the change in momentum. It follows that change in momentum ($mv_2 - mv_1$) = Ft. We may as well express this as ΔM = Ft (where ΔM is change in momentum and F and t are force and elapsed time, respectively).

It is worthy noting that the change in momentum is produced by an impulse. We can define the impulse of a constant force F acting for a time, Δt, as

Impulse = FΔt (i.e. Δ(mv) = FΔt)

Example

Let us now look at a momentum example:

A body weighing 800 N is uniformly accelerated from rest at 0.9 m/s^2. Calculate the momentum of the body after it has been moving for 5 s.

Solution:

Since the mathematical definition of momentum is M = mv, we can find momentum of the body by first finding its mass and multiply it by the final velocity:

To find the mass (m), we will use the second expression of Newton's Second Law of Motion which states that F = ma. Making m the subject of the formula yields

m = F/a

where "F" is the weight (force) of the object and "a" acceleration due to gravity (9.81 m/s^2)

Since 1 Newton (N) is equivalent to 1 $kg.m/s^2$, 800 N = 800 $kg.m/s^2$.

Hence mass, m = (800 $kg.m/s^2$) divided by 9.81 m/s^2

m = (800 $kg.m/s^2$) x s^2/9.81 m

m = 81.55 kg

Now, let us find the final velocity (V_2) of the object using motion Equation 2 (Eq 2):

$$V_2 = V_1 + at$$

where "V_2" is the final velocity, "V_1" initial velocity (= 0 since the body was initially at rest), "a" is acceleration (= 0.9 m/s^2) and t the time taken (= 5 s) for the body to accelerate from V_1 to V_2

Hence $V_2 = 0 + 0.9\ m/s^2 \times 5\ s$

$$V_2 = 4.5\ m/s$$

Since momentum, M = mv,

then momentum, $M = mV_2 = 81.55\ kg \times 4.5\ m/s$

= **366.98 kg.m/s**

4.4 Circular Motion

We have so far talked about motion in straight line. However, not all motion of objects occurs in a straight line. Motion also occurs in circular or curved paths. The motion of objects in a circular or curved path is referred to as **circular or angular motion**. For simplicity's sake, we will from now onwards assume that motion in a circular or curved path is uniform (i.e. the speed of the object is constant). The distance traveled in a specified direction by an object experiencing motion in a curved path is known as **angular displacement.**

In order to understand the concept of angular displacement, let us consider a particle attached to the end of a string and being rotated about some fixed point, O and the length

of the string being the radius of rotation, r. If the particle moves from point A to another point B, the particle moves through the arc length S and an angle θ. The angle, θ is the particle's angular displacement (see the Figure below).

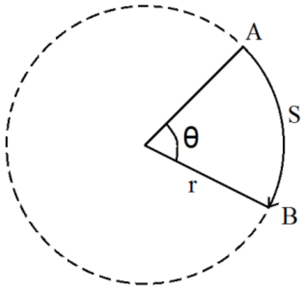


Figure 26. Angular displacement

The angular displacement, θ (i.e. the angle turned through by a body) when the radius (r) is equal to the length of the arc (S), is called a radian. By definition, θ the angular displacement is given in radians by the equation:

$$\theta = S/r$$

$$\text{Thus, } S = r\theta$$

Note that the radian is a dimensionless number (having no physical dimension or unit) since it is the ratio of the two lengths.

Now, since the circumference of a circle of radius r is 2πr, there are 2π radians in a complete circle.

Therefore 2π radians = 360 degrees

π radians = 180 degrees and

rad = 57 degrees 18 seconds

Here is a **definition of a radian:** "One radian is the angle subtended at the centre of a circle by an arc equal in length to the radius of the circle".

A particle experiencing angular motion also has velocity (i.e. it moves at a given speed (magnitude) in a particular direction) just like we saw under linear motion. The rate of change of a particle's angular coordinate (the angular displacement θ) with time is known as **angular velocity (ω)**.

Suppose θ changes from $\theta_f - \theta_o$ in time t. The average angular velocity (ω) of the particle can be defined as

$$\omega = (\theta_f - \theta_o)/t$$

The units of ω are rad/s or revolutions per minute (rev/min or rpm). Also

$$\omega \text{ (in rad/s)} = 2\,\pi f$$

where f is the frequency of rotation in rev/sec

You may recall that one of the causes of change in the velocity of a particle or object is a change in direction. Thus, a particle moving in a circular or curved path experiences a change in its angular velocity despite having a constant speed. The angular velocity change is due to the change in direction. Since a velocity change occurs over a given time interval, we may as well say that the particle in circular motion experiences some form of acceleration. The rate of change of angular velocity is known as **angular acceleration**. Hence, the **angular acceleration** (α) of a particle or object is the rate at which its angular velocity changes with time. It follows that if the angular velocity of a particle changes uniformly from ω_o to ω_f in time t, its angular acceleration (α) is

$$\alpha = (\omega_f - \omega_o)/t$$

whose units are rad/s^2 or rev/min^2 or rev/s^2.

So far, we have covered linear motion and a brief view of circular (angular) motion of particles. More details about circular motion are yet to come. In the meantime, let us compile some of the important equations used in solving problems related to these two kinds of motion. You will notice that the structure of linear motion equations is similar to those of their corresponding circular motion equations. This should make it much easier for you to remember both categories.

Linear	**Angular**
$V = \frac{1}{2}(V_o + V_f)$	$\omega = \frac{1}{2}(\omega_o + \omega_f)$
$S = Vt$	$\theta = \omega t$
$V = u + at$ $(V_f = V_o + at)$ (Eq 2)	$\omega_f = \omega_o + \alpha_t$
$V^2 = U^2 + 2as$ $(V_f^2 = V_o^2 + 2as)$ (Eq 5)	$\omega_f^2 = \omega_o^2 + 2\alpha\theta$

$S = ut + ½ at^2$ ($S = V_o t + ½ at^2$) (Eq 4)	$\theta = \omega_o t + ½ \alpha t^2$

Example

Let us now look at an example related to circular motion.

The body of a pendulum 90cm long swings through a 15cm arc. Find the angle θ, in radians and degrees, through which it swings.

You will recall that we defined the relationship between angular displacement (θ), the radius of the curve (r) and arc length (S). We will use this relationship here in finding the solution for this problem as follows:

$$S = r\theta \text{ or } \theta = S/r$$

From the question, S = 15 cm = 0.15 m and r = 90 cm = 0.90 m

Hence, θ = S/r = 0.15m/0.90m

$$\theta = \underline{\mathbf{0.167\ rad}}$$

Now, let us find θ in degrees:

To do this, we will apply the knowledge that we have that the circumference of a circle (or full revolution or 2πr; where r is radians) covers a 360 degree angle (i.e. there are 360° in the for every 2π radians). Therefore, θ in degrees can now be found as

$$\theta = (0.167 \text{ rad}) (360 \text{ deg}/2\pi \text{ rad})$$

$$\theta = \underline{\mathbf{9.55^o}}$$

4.1.10 Uniform Circular Motion

We will now have a more detailed look at real life circular motion problems in order to further our understanding of this kind of motion. To begin with, let us consider a car moving in a circular path (at a roundabout) at a constant speed v as illustrated in **Figure 28**.

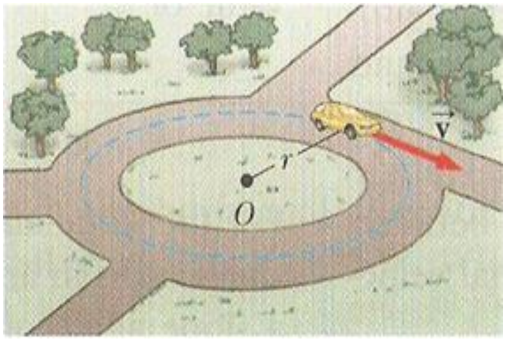


Figure 27. A car moving in a circular path at a constant speed

Even though the car (or any other object) moves with constant speed in a circular path, it still has acceleration. Acceleration depends on the change in velocity. As we saw earlier on in this unit (see **Figure 22 c**), velocity can change due to a change in direction despite the speed remaining constant. We also saw that the rate of change in velocity with respect to time is called acceleration. Hence the change in direction of the car moving in a circular path brings about a change in velocity. This change in velocity (due to change of direction) per unit time is the car's angular acceleration. According to Newton's First Law of motion, as discussed earlier on, there must be some external force acting on the car in order for it to maintain moving in the circular path. In other words, the acceleration that the car is undergoing must be a result of a force acting on the car so that it does not move in a straight line but remain in a circular path. This force is called **centripetal force**. Centripetal means **center seeking**. In other words, the force that is keeping the car on the circular path is acting towards the center (O) of the circle. Since the force is centripetal then the corresponding acceleration is referred to as **centripetal acceleration**. Centripetal acceleration is mathematically defined as

$a_c = v^2/r$

where a_c is centripetal acceleration; v is the linear speed of the car; and r the radius of the circular path.

Now, you can draw two vector diagrams using the radii of the circle $\underline{r}_i$ and $\underline{r}_f$ (b) and the velocities (c) at locations (A) and (B) as presented in **Figure 29**.

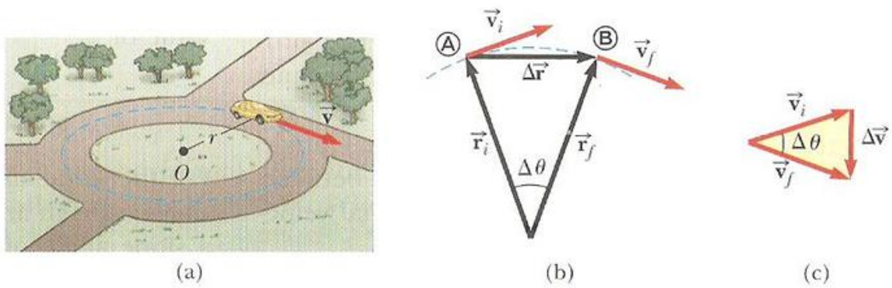


Figure 28. Vector diagram for motion in a circular path

In this **figure**, $\underline{r}_i$ and $\underline{r}_f$ are the initial and final radii, respectively, while $\underline{v}_i$ and $\underline{v}_f$ are the initial and final velocities at locations A and B, respectively. Since the direction of the radius vector is changing, the change in radius is $\underline{r}_f$ - $\underline{r}_i$ = $\underline{\Delta r}$. Similarly, the change in velocity is $\underline{v}_f$ -$\underline{v}_i$ = $\underline{\Delta v}$. You should know that it has been mathematically proven that the radii triangle and the velocity triangle are similar to each other. When you apply the principle of similar triangles, the ratio of $\underline{\Delta v}$ to v equals the ratio of $\underline{\Delta r}$ to r, i.e.,

$$\frac{|\Delta\vec{v}|}{v} = \frac{|\Delta\vec{r}|}{r}$$

Let us now assume that the rate of change of r (i.e. $\underline{\Delta r)}$ equals v (the speed of the car). This implies that the equation would now read

$$\frac{|\Delta\vec{v}|}{v} = \frac{v}{r}$$

If we make $|\underline{\Delta v}|$ the subject of the formula, $|\underline{\Delta v}| = v^2/r$. Therefore the rate of change in velocity per unit time gives us centripetal acceleration. Hence $a_c = v^2/r$ (= centripetal acceleration).

You should know that in many situations, it is convenient to describe the motion of a particle moving with constant speed in a circle of a radius r in terms of the period T, which is the time interval required for one complete revolution of the particle.
In the time interval T, the particle moves a distance of 2πr, which is equal to the circumference of the particle's path. Therefore, because its speed is equal to the circumference of the circular path divided by the period, or v = 2πr/T, it follows that

$$T = \frac{2\pi r}{v}$$

Let us consider a ball of mass m that is tied to a string of length r and is being whirled at constant speed in a horizontal circular path as shown in **Figure 30**.

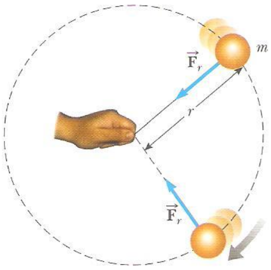


Figure 29. A ball whirled at a constant speed in a horizontal circular path

A force causing a centripetal acceleration acts toward the centre of the circular path and causes a change in the direction of the velocity vector. If that force should vanish, the object would no longer move in its circular path; instead, it would move along a straight-line path tangential to the circle (**Figure 31**).

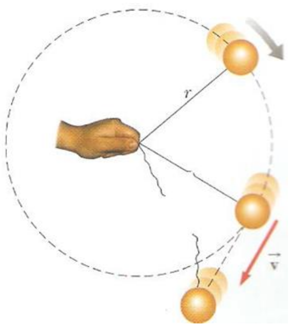


Figure 30. A ball released from a broken rope

Why does the ball move in a straight line? According to Newton's first law, the ball would move in a straight line if there were no force on it; the string, however, prevents motion along a straight line by exerting on the ball a radial force F that makes it follow the circular path. This force is directed along the string toward the centre of the circle as shown in **Figure 30** above. If Newton's second law is applied along the radial direction, we can relate the net force causing the centripetal acceleration to the acceleration as follows:

$$\sum F = ma_c = m\frac{v^2}{r}$$

Example

A small ball of mass m is suspended from a string of length L. The ball revolves with constant speed v in a horizontal circle of radius r as shown below (because the string sweeps out a surface of a cone, the system is known as a conical pendulum). Find an expression for v.

Solution:

We can illustrate the problem using the figure below.

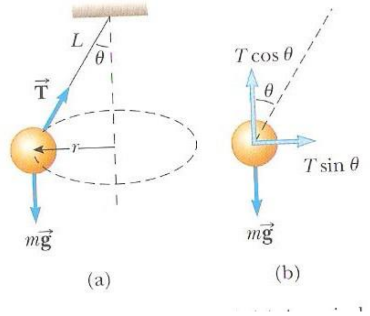


Figure 31. A ball suspended on a string

The ball does not accelerate vertically, i.e., it is in equilibrium in vertical direction.
It experiences centripetal acceleration in the horizontal direction, so it is modeled as a particle in uniform circular motion in this direction. Let θ represent the angle between the string and the vertical. In the free-body diagram, the force T exerted by the string is resolved into a vertical component T cos θ and a horizontal component T sin θ acting toward the centre of the circular path. From this, we can write the following equations bearing in mind that for the object to be in equilibrium vertically, summation of all the forces in the y direction must be equal to zero.

$$\sum F_y = T\cos\theta - mg = 0$$

From this expression, we can extract this equation:

T Cos θ = mg

Let us call this Equation (i)

Then let us define another equation for forces acting in the horizontal circular path in which the ball rotates:

$$\sum \boldsymbol{F}_x = \boldsymbol{T}\sin\boldsymbol{\theta} = \boldsymbol{m}\boldsymbol{a}_c = \frac{\boldsymbol{m}\boldsymbol{v}^2}{\boldsymbol{r}}$$

Similarly, we can extract the following equation:

$$T\sin\theta = \frac{mv^2}{r}$$

Then let us call this Equation (ii)

Next, let us divide Equation (ii) by Equation (i). This exercise will yield:

$$\frac{T\sin\theta}{T\cos\theta} = \frac{mv^2}{rmg}$$

Simplifying this equation yields the following since we know that sin θ divided by cos θ is tan θ:

$$\tan\theta = \frac{v^2}{rg}$$

We can now make v the subject of the formula and yield:

$$v = \sqrt{rg\tan\theta}$$

You will notice that the speed is independent of the mass of the ball since the final equation does not have mass (m) in it.

Example

Let us now look at some example related to motion of a car on a bend or curve. These examples will shed some light to us on why road curves are banked (raised on the outer part of the curve).

A 1500 kg car moving on a flat, horizontal road negotiates a curve as shown below. If the radius of the curve is 35.0 m and the coefficient of static friction between the tyres and dry pavement is 0.523, find the maximum speed the car can have and still make the turn successfully.

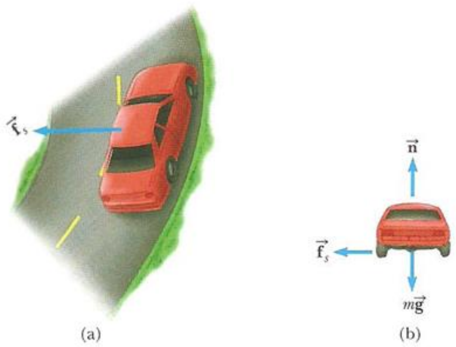


Figure 32. A car negotiating a bend

As we can see from the figure, (a) is an illustration of the car negotiating the curve while (b) illustrates the forces acting on the car. The car is vertically in equilibrium, hence the summation of the vertical forces ($\vec{n}$ and $m\vec{g}$) equals zero. This statement can be presented mathematically as $\sum F_y = 0$and $\vec{n}$ - $m\vec{g}$ = 0. It follows from the latter equation that $\vec{n} = m\vec{g}$.

We saw earlier on that $v = \sqrt{rg \tan\theta}$ and you will recall that when we talked about friction we saw that $\tan\theta = \mu$, the friction factor. Therefore, taking the friction factor of the surface given in the question above as μ_s we can now solve the problem as follows:

$$\Sigma Fy = 0 \rightarrow n - mg = 0 \rightarrow n = mg$$

$$v_{max} = \sqrt{\mu_s gr}$$

$$\left(= \sqrt{\frac{\mu_s nr}{m}} = \sqrt{\frac{\mu_s mgr}{m}}\right)$$

$$v_{max} = \sqrt{(0.523)(9.8m/s^2)(35.0}$$

$$v_{max} = 13.4m/s$$

Example

Here is an example on a banked roadway:

A civil engineer wishes to redesign the curved roadway in the previous example in such a way that a car will not have to rely on friction to round the curve without skidding. In other words, a car moving at the designated speed can negotiate the curve even when the road is covered with ice. Such a ramp is usually banked, which means that the roadway is tilted toward the inside of the curve. Suppose the designated speed for the ramp is to be 13.4m/s and the radius of the curve is 35.0 m. At what angle should the curve be banked?

Solution:

On a level (unbanked) road, the force that causes the centripetal acceleration is the force of static friction between the car and road as was the case in the preceding example. If the road is banked at an angle θ as in the figure below, however, the normal force n has a horizontal component toward the centre of the curve. Because the ramp is to be designed so that the force of static friction is zero, only the component $n_x = \sin\theta$ causes the centripetal acceleration.

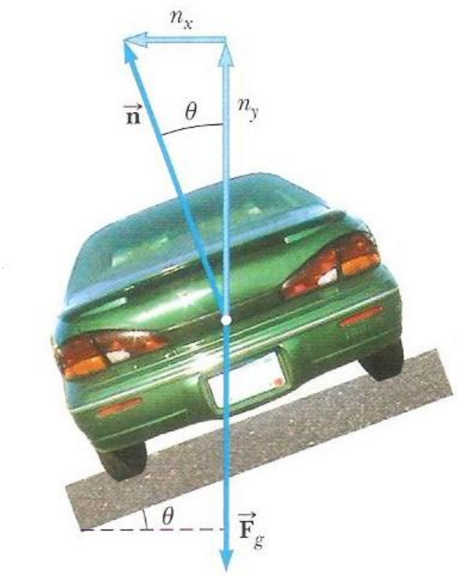


Figure 33. A banked roadway

From the figure above, we see that using the triangle of forces created in this figure yields the following relationship for the horizontal force (n_x):

$$\sin\theta = \frac{n_x}{\vec{n}}$$

hence, $\vec{n}\sin\theta = n_x$

You will notice that n_x is the centripetal force (i.e. the force keeping the vehicle in the circular path). From the definition that we saw earlier on, centripetal force is defined as $\frac{mv^2}{r}$. Therefore, $\vec{n}\sin\theta = \frac{mv^2}{r}$ (Eq. i)

Similarly, we can define a relationship for the vertical forces as follows:

$$\cos\theta = \frac{n_y}{\vec{n}}$$

and $\vec{n}\cos\theta = n_y$

but n_y is the weight of the vehicle, mg. Hence, $\vec{n}\cos\theta = mg$ (Eq. ii)

Now, let us divide equation i by equation ii and this will yield:

$$\tan\theta = \frac{mv^2}{rmg} = \frac{v^2}{rg}$$

Therefore, $\tan\theta = \frac{v^2}{rg} = \frac{(13.4m/s)^2}{(35.0m)(9.81m/s^2)}$

$$\tan\theta = 0.03903$$

$$\theta = \tan^{-1}(0.03903)$$

$$\theta = 27.6^o$$

The result implies that the curve must have an angle of banking of 27.6^o.

Notes:

1) From the solution obtained above (in the last part of the solution), we can see that the angle of banking is independent of the mass of the vehicle.
2) If the car rounds the curve at a speed greater than 13.4 m/s, it has to depend on the friction to keep it from sliding up the bank to the right.
3) If the car rounds the curve at a speed less than 13.4 m/s, friction is needed to keep it from sliding down the bank (to the left).

4.1.11 Projectiles

A projectile is an object which is given an initial velocity (x) in the horizontal (x) direction and is then allowed to move under the action of gravity. If we neglect air resistance, horizontal motion of a projectile is uniform while the vertical motion is uniformly accelerated. The path of a projectile is parabolic and is called a TRAJECTORY. The angle of projection and initial speed determine the range (R) of a projectile. We can define the range of a projectile as the horizontal distance covered by the projectile from its initial to final location. If we neglect air resistance, the maximum range of the projectile will be attained with an angle of 45^o. Since the object's position is changing both vertically and horizontally, we can define the vertical and horizontal velocity components of the object's motion. If the initial velocity of the projectile is V_o then its initial vertical (Y) velocity component is V_{oy} and initial horizontal (X) velocity component V_{ox}.

Let us now consider a projectile that is fired horizontally at an elevation y as presented in the figure below.

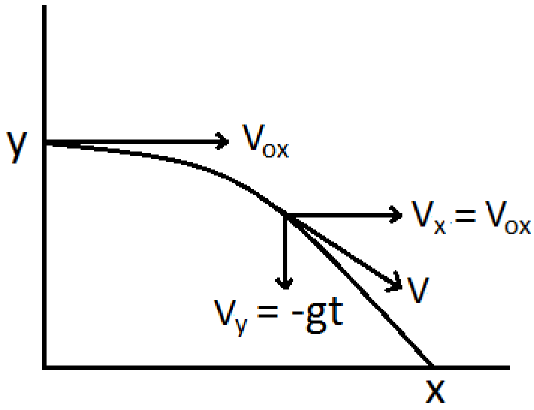


Figure 34. Projectile trajectory

The scenario in the figure also represents the last half of a trajectory for a projectile that has been fired at an angle with respect to the horizontal plane as we will see in the next figure. In the meantime, let us dwell on the figure above by looking at the vertical (Y) and horizontal (X) components of the velocity vector.

X component:

It is worthy for us to note that the horizontal velocity component of the projectile will remain constant if air resistance is negligible. This implies that the object is experiencing zero acceleration horizontally (i.e. a = 0). It follows that the X component of the velocity vector will be $V_x = V_{ox}$ at any given time during the motion of the object. Here, we are applying motion equation 2 whereby instantaneous velocity (or final velocity for a given time instance), V_x equals initial velocity (V_{ox}) plus zero (0) times time (t): $V_x = V_{ox} + 0t = V_{ox}$). From this, we can find the horizontal displacement of the object at any time instance as the product of the velocity (V_{ox}) and elapsed time. We can mathematically represent this as $x = V_{ox}t$.

Y component:

Again, if we assume that air resistance is negligible, the vertical component of the velocity of object will only be acted upon by gravitational force. The gravitational force produces acceleration (i.e. the object is experiencing acceleration due to gravity). This acceleration is towards the center of the earth and we can designate it negative since it is acting downwards (i.e. acceleration = -g). You will recall that velocity is a product of acceleration and time (V = at). Hence, we can express the impact of gravity on the vertical component of the object's velocity as -gt. This implies that instantaneous velocity for the horizontal motion of the projectile can be defined as the sum of the initial velocity (V_{oy}) and the influence of gravity (-gt). Thus, $V_y = V_{oy} – gt$. Similarly, we can determine vertical displacement of the object at any time instance as the product of the instantaneous velocity ($V_{oy} – gt$) and elapsed time (t): $y = V_{oy}t – gt^2$.

We can now summarize what we have discussed as follows:

Acceleration: $a_x = 0$; $a_y = -g$

Velocity: $V_x = V_{ox}$; $V_y = V_{oy} – gt$

Displacement: $x = V_{ox}t$; $y = V_{oy}t – gt^2$

Now, let us look at a projectile fired with an initial velocity V_o at an angle θ with respect to the horizontal as represented in the figure below.

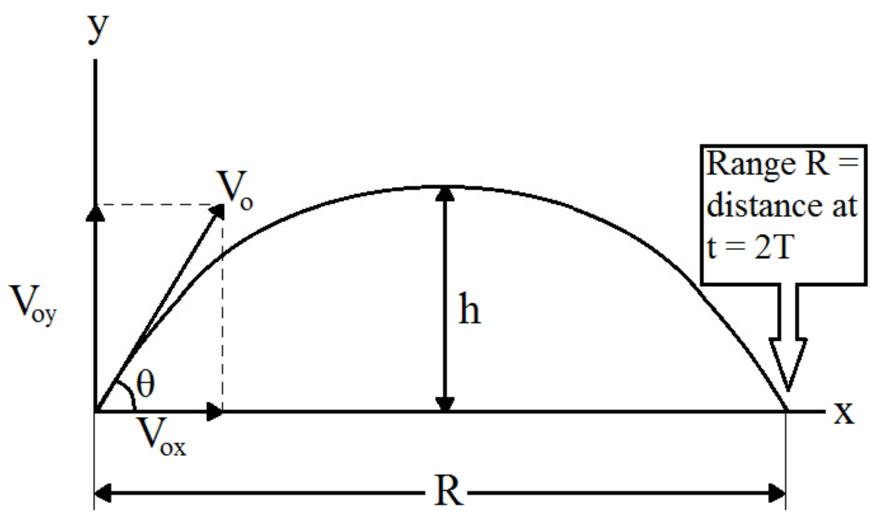


Figure 35. The path of a projectile fired with an initial velocity V_o at an angle θ with respect to the horizontal

Figure 36 shows the path followed by a projectile fired with an initial velocity Vo at an angle θ with respect to the horizontal. We can define the horizontal and vertical components of the velocity as follows:

$$V_{ox} = V_o \cos\theta$$ (Equation 1)

$$V_{oy} = V_o \sin\theta$$ (Equation 2)

We previously saw that

$$x = V_{ox} t$$ (Equation 3)

$$y = V_{oy} t - gt^2$$ (Equation 4)

Hence, the velocity components at any instant time are given by

$$V_x = V_{ox} = V_o \cos\theta$$ (Equation 5)

$$V_y = V_{oy} - gt = V_o \sin\theta - gt$$ (Equation 6)

Let us now use these equations to find the maximum height to which the particle will rise and the overall range of the projectile along the horizontal surface. In order to find the maximum height, we calculate the value of Y when the upward component of the velocity has decreased to zero, that is, Vy = 0 when Y = h. Let t = T when this occurs. If we substitute Vy = 0 and t = T into Equation 6:

$$V_y = V_{oy} - gt$$ Equation 7

$$0 = V_{oy} - gT$$ Equation 8

$$T = \frac{V_{oy}}{g}$$ Equation 9

Now let us use equation

$$h = V_{oy}T - \frac{1}{2}gt^2$$ Equation 10

$$h = V_{oy}\left(\frac{V_{oy}}{g}\right) - \frac{1}{2}g\left(\frac{V_{oy}}{g}\right)^2$$ Equation 11

$$h = \frac{V_{oy}^2}{g} - \frac{1}{2}\frac{V_{oy}^2}{g}$$ Equation 12

or $$h = \frac{1}{2g}V_{oy}^2 = \frac{1}{2g}V_o^2 \sin^2\theta$$ Equation 13

Equation 13 gives the maximum height h in terms of the initial velocity. In order to find the horizontal range R we us the fact that the total flight requires a time that is twice the time necessary to each maximum height. That is we set X = R when t = 2T. Then using Equation 3 we find

$$R = V_{ox}2T$$ Equation 14

Substituting for T where $T = \frac{V_{oy}}{g}$, we have

$R = \frac{2V_{o\,x} V_{o\,y}}{g}$, and making use of equation 1 and 2 substituting for V_{ox} and V_{oy} gives

$$R = \frac{2(V_0 \sin\theta)(V_0 \cos\theta)}{g}$$

Therefore, $R = \frac{2V_O{}^2 \sin\theta \cos\theta}{g}$ Equation 15

From trigonometry, 2 sinθ cosθ = sin 2θ. Therefore, we have

$R = \frac{V_0^2 \sin 2\theta}{g}$ Equation 16

From Equation 16, we see that the maximum range, R_{max} will occur when $\theta = 45^o$. This is because sin 2θ becomes sin $90^o = 1$. Therefore,

$R_{max} = \frac{V_O{}^2}{g}$ at $\theta = 45^o$ Equation 17

Let us now summarize the kinematic equations for projectile motion:

Table 2. Kinematic equations for projectile motion

Horizontal Motion	Vertical Motion
$V_x = V_{ox}$	$V_y = V_{oy} – gt$
$x = V_{ox}t$	$y = V_{oy}t – gt^2$
	$V_y^2 = V_{oy}^2 + 2gy$

Example

An artillery piece is pointed upward at an angle of 35^o with respect to the horizontal and fires a projectile with a muzzle velocity of 550 m/s. If air resistance is not important

a) To what height will the projectile rise?

b) What will be its range?

Solution:

a) Let us use Equation 13 to find the height h:

$$h = \frac{1}{2g} V_{oy}^2 = \frac{1}{2g} V_o^2 \sin^2 \theta$$

$$h = \left(\frac{550m}{s}\right)^2 \times \frac{s^2}{2 \times 9.81m} \times \sin^2 35^\circ$$

$$h = 5072$$

b) We can find the range of the projectile using Equation 16 as follows:

$$R = \frac{V_o^2 \sin 2\theta}{g}$$

$$R = \left(\frac{550m}{s}\right)^2 \times \frac{s^2}{9.81m} \times \sin 70^\circ$$

$$R = 28976m$$

Unit Sumary

In this unit, we have covered basic principles of motion. Motion is one of the integral parts of mechanics. Any motion experienced in animals, water bodies, farm machinery, light, etc. utilizes motion principles discusses in this unit.

Unit est

1. A farm truck initially traveling at a speed of 90 km/h decelerates constantly to a speed of 10 km/h in 3 minutes.
   a. What is its deceleration?
   b. How far must it travel to attain the final speed?
2. A ball of mass 400 g is attached to the end of a cord 120 cm long. The ball is whirled in a horizontal circle. The maximum speed at which the ball can be whirled before the cord breaks is 15 m/s. Find the maximum tension (N) that the cord can withstand. Assume that the string remains horizontal during the motion.

Suggested Answers to the Unit Test

Here are the suggested answers for the Unit Test:

1. a) 0.1236 m/s or -0.1236 m/s or 0.4 km/hr or -0.4 km/hr;
   b) 2500 m (or 2.5 km)

2. 75N

UNIT 5: WORK, ENERGY AND POWER

5.0 Introduction

In everyday language, the word work is used to describe any activity in which muscular or mental effort is exerted. In physics the word work has a special meaning. We say work has been done only when force acting on a body produces motion in the direction of the force (or in the direction of the component of the force). The speed at which work is done indicates the power of the body doing work. Time is an important aspect of power. A body that has capacity to do work is said to possess energy. Energy exists in different forms. Work, energy and power are related to each other and they form three fundamental principles of physics that we are going to deal with in this unit.

5.1 Objectives

By the end of this unit, you should be able to:

- Differentiate work done by a force from work done by torque.
- State different forms of energy and their examples
- State the law of conservation of energy
- Derive equations and units of power

Key terms

Ensure that you understand the key term phrases used in this unit as listed below.

- Work done
- Energy
- Power
- Work-Energy Theorem
- Strain Energy

5.2 Work done by a force

Work done is the transfer of energy from one form to another by the means of the action force applied over a distance.

Work Done (W) = $Fd\cos\theta$ 5.1

Where F is force, d is distance and θ is the angle between the direction of motion and the line of application of the force.

When $\theta = 0$, work done = Fd, and when $\theta = 180$, work done = - Fd.

5.2.1 **Nature of work done**

Work done can either positive or negative depending on the value θ.

1. Positive work
   Work done is positive when $\theta \leq 90°$, in this case $\cos\theta$ is positive. Under this circumstance, the component of force moves in the direction of object's displacement. Some examples of positive work are:
   a. Free falling body under the influence of gravity
   b. Stretching a spring, in this case the stretching force and the displacement of the spring are in the same direction, $\theta = 0$ and W=Fd
   c. The pulling of a cart by a horse on a level ground

2. Negative work
   When $\theta > 90$, $\cos\theta$ is negative and the work done is negative. Under this condition there is a component of force that moves opposite to the direction of object's displacement. We have some examples of negative work.
   a. Work done by friction force, $\theta = 0$, $\cos\theta = -1$
   b. Work done by electrostatic forces of the same charge
   c. Work done by gravity on a body moving vertically upwards

3. Zero work done
   If $\theta = 90°$, $\cos\theta = 0$ and no work is done by the force on the body. We also have a zero work done when either F or S is zero. Examples of zero work:
   a. A woman carrying some load on his head moving on a level road, $\theta = 90$

b. A man pushing the wall, S or d = 0

c. A body moving in a circle with the help of the string, S = 0

Example

An object is being pulled along the ground

5.2.2 **Work done by torque**

You may recall from unit 3 that torque is given as the product of force and distance. Using this relationship we can derive a formula for work done by torque.

$$\tau(torque) = Force(F) \times Distance(r) \qquad 5.2$$

but work (W) = F.S

Consider circular motion: $\theta = \frac{S}{r}$, making S subject we get S = rθ

Then W = Frθ 5.3

Substituting equation 5.2 into 5.3 we get;

$$W = \tau\theta \qquad 5.4$$

Work done by torque is given by $\tau\theta$.

Activity 1.

1. A gardener moves a lawn roller through a distance of 50 m with a force of 50N inclined at an angle of 60° to the direction of motion. Calculate his wages if he is to be paid K200 for doing 25J of work.
2. How much work is needed to push a 1250kg cart 11.5 m up at 13.5° incline at constant velocity?

   i. Ignore friction

   ii. Assume the coefficient of friction is 0.09

5.3 Energy

Energy is the measure of a change imparted into a system. We may also define energy as the capacity to do work. The magnitude of energy depends on the capacity of a body to do work. The greater the capacity a body has to do work, the greater the energy it has. Energy exists in different forms such as mechanical energy, heat energy, sound energy, light energy etc.

FORMS OF ENERGY

a. Mechanical energy
   i. Kinetic energy
   ii. Potential energy
      - Gravitational potential
      - Elastic potential
b. Heat energy
c. Internal energy
d. Electrical energy
e. Chemical energy
f. Nuclear energy

5.3.1 **Potential energy**

The energy possessed by a body due to its position or configuration (shape or size) is referred to as potential energy.

Potential Energy (PE) = mgh 5.5

5.3.2 **Kinetic Energy**

Kinetic energy is the energy possessed by a body in motion. We use kinetic energy in most of our daily activities; for example, a hammer driving nails into a piece of wood possesses kinetic energy.

We derive the equation for kinetic energy from the equation equations of motion studied in unit 4. Consider the equation:

$v^2 - v_0^2 = 2aS$ If an object begins from the rest $v_0^2 = 0$

then, $v^2 - 0 = 2aS$ or $v^2 = 2aS$, If we make S subject

$$S = \frac{v^2}{2a} \qquad 5.6$$

But W = FS, at constant acceleration F = ma (Newton's second law)

Now we have W = maS 5.7

Substituting equation v into equation vi, we have

$$W = ma\frac{v^2}{2a} = \frac{1}{2}mv^2$$

Since work done W = kinetic energy gained by the body

$$K = \frac{1}{2}mv^2 \qquad 5.8$$

From equation iii we see that kinetic energy of a moving body is directly proportional to the mass and square of the velocity of the body.

5.3.3 **Strain Energy**

We define strain energy as the amount of energy stored in a wire. In other words it is half of the product of stress and strain or the energy per unit volume.

Strain Energy = ½ strain × stress 5.9

But $Stress = \frac{(Force)F}{(Area)A}$, $Stress = \frac{(elongation)e}{(Length)L}$

Strain Energy = $\frac{1Fe}{2AL}$ 5.10

Fe = Energy and AL = Volume

$$\frac{Energy}{Unitvolume} = \frac{1}{2}\frac{Fe}{AL} = \frac{1}{2}Stress \times Strain$$

5.4.3 **Transformation of Energy**

In all physical processes, energy is transformed from one form to another. In real life experiences we have a number of examples, when you take food into your stomach, chemical energy from the food is converted into mechanical and electrical energy which allows the body to move. Below are some few examples:

1. The sun transforms nuclear energy into ultraviolet, infrared, and gamma energy and all forms of electromagnetic energy.
2. When lighting strikes a tree, electrical energy is converted to electrical energy.
3. When you stretch a catapult (legeni) to fire a stone, energy of the catapult is turned into strain energy of the rubber bands. Upon releasing the pull, strain energy is transferred to stone that gains kinetic energy and then moves at high speed. Kinetic energy of the stone is changed to sound and heat energy when the stone hits the target.

5.4.3 **Conservation of energy**

One important aspect of energy is that it is a conserved quantity. In any process energy may be transferred from one object to another or from one form to another, but the total energy neither increases nor decreases. For example, when you throw a stone from your hand, you are simply changing potential energy to kinetic energy then to sound and heat energy.

The law of conservation of energy states that, ***the total energy neither increases nor decreases. Energy can be transformed from one form to another; and transformed from one body to another but the total amount remains the same*.**

According to the law of conservation of energy, the amount of energy in your village, district and the whole universe is the same now as it was millions of years ago.

5.4.4 **Work-Energy Theorem**

After we have discussed both work done and kinetic energy it will be helpful to explore how these two are related. Work energy theorem simply describes the relationship between work and kinetic energy.

The theorem stipulates that, *"the work done by the net force acting on a body is equal to the change in the kinetic energy of the body."*

Mathematically, we can represent this theorem as:

Net Work done = Change in kinetic energy

$$W = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2 \qquad 5.11$$

From this theorem we may note the following:

- If net force does positive work on a body, the K.E. of the body increases.
- If net force does negative work on the body, the K.E of the body decreases
- If work done on a body is zero, K.E. of the body remains the same.

Activity 2.

Discus in groups of three if the following statements are true or false;

1. The kinetic energy of a body at rest is zero
2. When m and v^2 are both positive then kinetic energy of a body is always positive.
3. Kinetic energy is a vector quantity
4. Kinetic energy of a moving body depends on its speed and not on the direction in which it is moving.

5.4 Power

Power is the rate of doing work. The faster a given amount of work is done by an agent, the greater is its power and vice-versa.

$$Power(P) = \frac{Workdone(W)}{Time(T)} \qquad 5.12$$

We can also obtain another useful relation for power if we bring in the equation for work done into the equation of power.

$W = \vec{F} \times \vec{S}$ then,

$$P = \frac{W}{t} = \frac{\vec{F}.\vec{S}}{t}$$

but $\frac{\vec{S}}{t} = \vec{v}$

$$\therefore P = \vec{F}.\vec{v} \qquad 5.13$$

Equation (xii) is referred to as **instantaneous power**.

Sometimes we define power interms of energy, thus power is the rate of dissipating energy. Let us first consider potential energy (PE).

$$Power = \frac{PE.}{t} = \frac{mgh}{t}$$

$$P = \frac{mgh}{t} \qquad 5.14$$

It seems we have achieved to derive an equation of power from potential energy. Let us now kinetic energy.

$$Power = \frac{KE}{t} = \frac{1}{2}.\frac{mv^2 - mv_0^2}{t}$$

$$P = \frac{m(v^2 - v_0^2)}{2t} \qquad 5.15$$

In calculations we are at liberty to choose any of these four equations (xi, xii, xiii and xiv) depending on the given information.

Example

A LUANAR student wants to pump oil from the tanks of a ship to a storage tank on land 45 m higher in elevation

5.4.1 Units of power

The SI unit of power is Watt or J/s, thus the power of an agent is 1W if it is doing 1J of work in 1s. Sometimes we use bigger units of power such as kilowatt (kW), megawatt (MW) or horse power (h.p) to simplify calculations.

1 kW = 1000 W; 1 MW = 10^6 W ; 1 h.p. = 746 W

5.4.2 Power transmission and torque

Activity 3.

1. A 1 h.p. electric motor operates a pump continuously. How much work does the motor perform in one day?
2. A farmer cycles up a 6° hill at a steady speed of 7km/h. if the mass of the farmer and his bicycle is 75 kg, find the power output of the farmer.

5.5 Practice activity

Draw three vectors forming a right-angled triangle, and calculate the resultant vector. Change the order in which the vectors were joined whilst maintaining the right-angled triangle and calculate the resultant of the new triangle.

5.6 Unit summary

Having covered work, energy and power, we can now conclude that power is derived from both energy and work done. Power is the rate of doing work or releasing energy.

Unit Test

1. Schmedrick takes his 1800 kg pet rhinoceros, Gertrude, ice-skating on a frozen pond. While Gertrude is coasting past Schmedrick at 4 m/s, Schmedrick grabs on to her tail to hitch a ride. He holds on for 25 m. Because of friction between the ice and Schmedrick, Gertrude is slowed down. The force of friction is 170 N. Ignore the friction between Gertrude’s skates and the ice. How fast is she going when he lets go?
2. A child pulls a toy of 2m across the floor by a string, applying a force of constant magnitude 0.8N. During the first metre, the string is parallel to the floor. During the second metre, the string makes an angle of 30° with the horizontal. What is the total work done by the child on the toy? [1.5]

3. Oil is pumped from the tanks of a ship to a storage tank on land 45 m higher in elevation. What is the power required to pump 20000 litres of oil per hour? Given that 1 litre of oil has a mass of 0.8 kg. [1960 W]
4. A ball of mass 400 g is thrown upwards at a speed of 5 ms^{-1}. (g = 9.81 N kg^{-1}). Calculate
   a. The ball's kinetic energy as it is released.
   b. The ball's maximum gain in potential energy.
   c. The ball's maximum height.
5. What is the resistive force on a cyclist who has leg muscles of power 200 W each and who reaches a top speed of 10 ms^{-1} on a level road?

Suggested Answers to unit Activities

Activity 1

1. K10 000
2. a). 3.29×10^5 J b). 4.5×10^5 J

Activity 2

1. True
2. True
3. False
4. True

Activity 3

1. 18 kWh
2. 300 W

Unit Test.

1. 3.36 ms^{-1}
2. 1.5J
3. 1960 W
4. a). 5J b). 5J c). 1.27m
5. 40N

UNIT 6: SIMPLE MACHINES

6.0 Introduction

Welcome to unit 6! In unit 5, we discussed work, power and energy. In this unit we shall the concepts from unit 5 and the other preceding units in the operation of simple machines. Most of us still use knives for cutting vegetables, axes for chopping firewood, a wheelbarrow for carrying bricks, a bottle opener to open soft drinks and a hammer to fix or remove nails. All these are examples of simple machines. We use them to do work easier and faster than we could have performed without them. In this case, we can see that using a machine gives an advantage than without using a machine. For example it is easier and more efficient to open a bottle of coca-cola using a bottle opener than using human teeth. We are more efficient when we do work using machines.

6.1 Objectives

By the end of this unit should be able to:

- List three categories of simple machines
- Define mechanical advantage
- Differentiate ideal mechanical energy from actual mechanical energy
- Evaluate efficiency and velocity ratio of different categories of simple machines

Key terms

- Actual Mechanical Advantage
- Ideal Mechanical Advantage
- Velocity Ratio
- Efficiency
- Gears
- Pulleys

6.2 Machines.

We are all surrounded by simple machines beginning from a human body. Most of our activities are done with the help of machines. For instance, activities like eating, walking and jumping are aided by machines (levers) in a human body. A machine is usually defined as a device that makes work easier by applying the following:

a. Transferring force from one place to another
b. Changing the distance or speed of the applied force or
c. Changing the magnitude of the applied force
d. Changing the direction in which you apply force

From these four factors above, we may define a machine as a device that changes the magnitude, direction, or the mode of application of a force so as to achieve some advantage. Simple machines have few or no movable parts.

We normally classify simple machines into three categories:

i. Levers
ii. Inclined plane (a ramp)
iii. Hydraulic press

Activity 1

Categorize the following machines in their appropriate categories (Lever, inclined plane, Hydraulic or screw and axle); Screwdriver, an axe, a cooking stick, knife, doorknob, door steps, zipper, ox-cart, bloom and a hydraulic jack lifter.

6.3 Mechanical Advantage

Our major purpose in applying machines (panga knives, an axe, wheelbarrow, e.t.c) in doing work is to simplify the work by using little effort on a heavy load. In other words the advantage that a machine gives is the utilization of a small effort to lift large loads. This principal in machines is termed as mechanical advantage.

Mechanical advantage is the ratio of the output force (load) to the input force (effort). It is the factor by which a machine multiplies applied force.

Mechanical Advantage (MA) = $\frac{Load}{Effort}$ 6.1

We may further subdivide mechanical advantage into two; actual mechanical advantage and ideal mechanical advantage.

Actual Mechanical Advantage (AMA) is the force that a machine can multiply while subtracting loses from the machine having to overcome friction. It is basically a force ratio and it's given by:

Actual Mechanical Advantage (AMA) = $\frac{Load}{Effort}$ 6.2

Ideal Mechanical Advantage (IMA) is the theoretical mechanical advantage of an ideal machine where there is no friction force. We sometimes refer ideal mechanical advantage to movement ratio or velocity ratio, which is simply the ratio of the distance moved by the effort to the distance moved by the load. We present it as:

Ideal Mechanical Advantage (IMA) = $\frac{Effort-dis\tan ce}{Load-dis\tan ce}$ 6.3

NOTE: The greater the mechanical advantage, the greater will be the load that can be moved for a given effort.

Activity 2

After looking at mechanical advantage of simple machines, can you explain the major difference between actual mechanical advantage and ideal mechanical advantage?

6.4 Velocity Ratio

You may recall, that we have already come across this word under ideal mechanical advantage. Velocity ratio like ideal mechanical advantage is the ratio of the distances moved by the effort and load. We can present it the way we have done in equation 6.3.

$$\text{Velocity Ratio (VR)} = \frac{Effort-distance}{Load-distance} \qquad 6.4$$

Activity 3

Calculate the velocity ratio of a 2.5 m lever with a 50kg load placed 0.5m from the pivot.

6.5 Efficiency

Our experience with the machines that we use in day to day lives its that the output energy is always less than the input energy. This is a result of loss of some energy due to friction. In most cases this energy is transformed to heat.

In this case, work done on a machine which we know from unit 5 that it is just the transferred energy, is given as;

Work done on machine = work done by machine + work done in moving the parts of the machine + work done in overcoming friction.

We increase the efficiency of a machine by the ratio of the useful work got out of the total work put in.

In the context of simple machines, the ratio of output work to the input work is called efficiency. However, we are not limited to this definition alone, we cay use any of the relationships (equations) above to work out efficiency. Below are some of the equations we may use to calculate efficiency.

Efficiency = $\frac{Work-out}{Work-in}$ or 6.5

Efficiency = $\frac{Power-out}{Power-in}$ or 6.6

Efficiency = $\frac{AMA}{IMA}$ or 6.7

Efficiency = $\frac{MA}{VR}$ 6.8

Activity 4

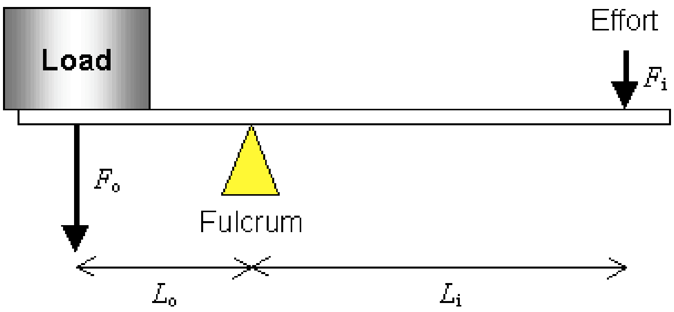
Explain why is it not possible to have a machine with 100% efficiency in real situation?

6.6 Categories of simple machines

6.6.1 **Levers**

What usually comes in your mind when you hear the word levers? Most people think of a see saw. This category of simple machines is commonly used everywhere and every day. A lever is a rigid bar that rotates around a fixed point called the fulcrum. The bar may either be straight or curved. Levers have three basic parts, effort, load and the fulcrum. We can classify levers into three classes by rearranging the orientation of these three (effort, load and fulcrum) basic parts.

Levers make lifting weight easier by using a fulcrum to redirect force over a longer distance. The following are some of the examples of levers that we use or we have ever used; see-saw, bottle opener, broom, hammer claw, screwdriver, crowbar, crane arm, fishing pole, etc.



Mechanical Advantage and Velocity Ratio of levers

Mechanic advantage of levers is given by equation 6.1, where as velocity ratio is given by equation 6.4. In this case effort and load's distance are with reference from the fulcrum.

Example 6.1

Determine the force required to support the 90N on a lever as shown below. Evaluate the values of IMA, AMA and efficiency of the machine?

Solution:

i. We first of all apply the principal of moments: 4 x F = 2 X 90, solving for F, we have; F = 45N
ii. Lets apply equation 6.3 to find IMA; IMA = 4/2 = 2
iii. Like wise lets apply equation 6.1 to find AMA; 90/45 = 2
iv. Finally for efficiency we will consider equation 6.7; 2/2

Velocity Ratio of greater than one means that the input distance (or effort) to move a load will be greater than the output distance of the load and if is less than one its visa versa.

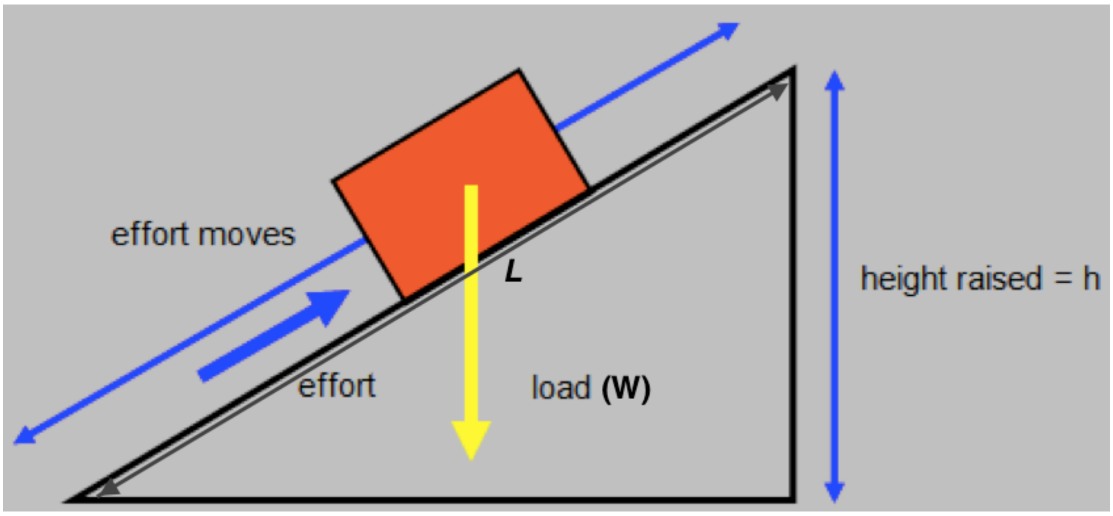
Activity 5

In this section we have discussed that levers are categorized into different classes, State three classes of levers with three examples on each.

6.6.2 **Inclined planes**

An inclined plane is sloping surface.

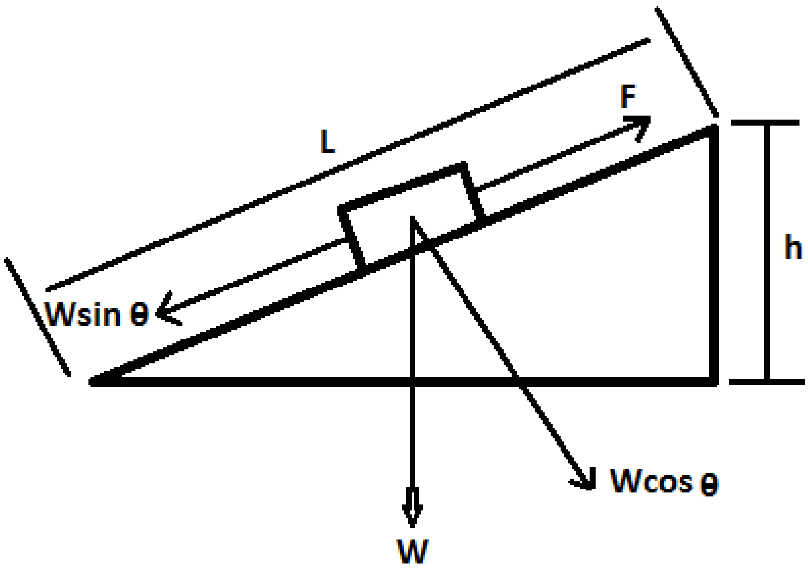
An inclined plane can be used to alter the effort and distance involved in doing work, such as lifting loads. The trade-off is that an object must be moved a longer distance than if it was lifted straight up, but less force is needed.



We calculate work done, using the usual equations in chapter 5

Work done by load (output) = load (weight) x height raised (h)

Work done by effort = Effort (Force) x ramp length (l)



Mechanical Advantage and velocity ratio of inclined planes

Mechanical advantage calculated using equation 6.1. Velocity Ratio is given by the ratio of length of the lamp to the height of the ramp.

$$VR = \frac{Ramp..length}{Ramp..height} \quad 6.9$$

If we have an ideal machine; $\frac{weight(w)}{Force(F)} = \frac{height(h)}{length(l)}$

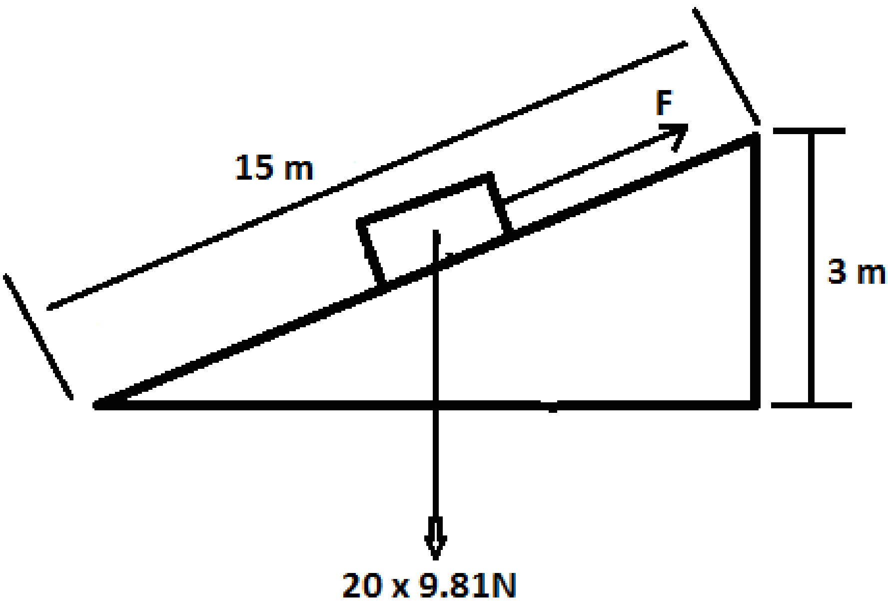
Example

An inclined plane 15m long rises 3m. Calculate the following:

a. The force F parallel to the plane required to slide a 20kg box up the plane if friction is neglected.
b. The IMA of the plane

Solution

Draw a diagram to have a pictorial presentation of the problem.

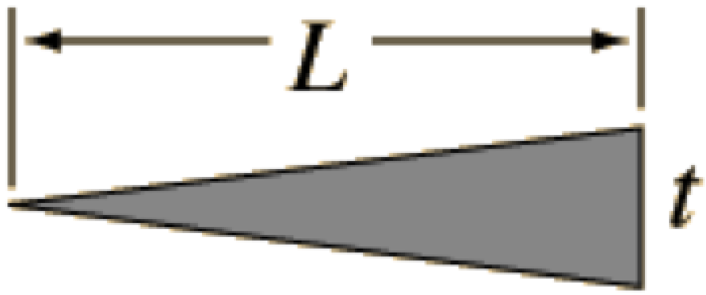


a. F x 15 = 20 x 9.81 x 3 = 39 N

b. IMA = 15/3 = 5.0

6.6.2.1 Wedge

A wedge is an example of an inclined plane. It can be used to raise a heavy load over a short distance or to split a log.



Mechanical Advantage and Velocity Ratio

We calculate the velocity ratio (IMA) of a wedge by taking the ratio of depth of penetration (L) to the separation of wedged surface (t).

$$VR = IMA = \frac{L}{t} \qquad 6.10$$

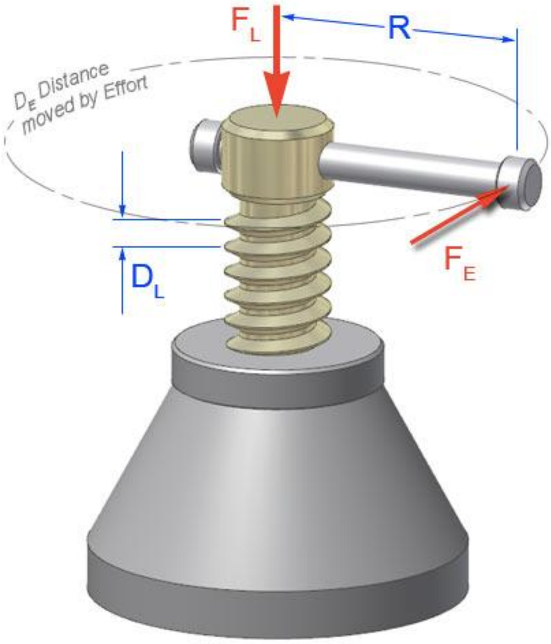
The ideal mechanical advantage (IMA) of a wedge depends on the angle of the thin end. The smaller the angle, the less the force required to move the wedge a given distance through a log. At the same time, the amount of splitting is decreased with smaller angles.

6.6.2.2 Screw

A screw is a variation of an inclined plane (or a wedge) in which the incline is wrapped around a shaft. The *pitch* of a screw is the distance between its threads (D_L), and this is the distance that the screw will advance with one complete rotation (D_E). To calculate to *IMA* you need to know the pitch, but you also need to know the length of the lever arm (R) being used to turn the screw. This lever arm could be the radius of a screwdriver handle, of the length of the handle on a vise or a C-clamp. This will allow you to calculate how far the screw will advance for a given lever movement.

Mechanical Advantage and Velocity Ratio

We define mechanical advantage for a screw using the same equation 6.1, but here load is F_L and effort is F_E. Velocity ratio is found by considering the ratio of circular distance moved by the lever (D_E) to the to the pitch (D_L).



$$VR = \frac{D_E}{D_L}$$

But $D_E = 2\pi R$

$$\therefore\ VR = \frac{2\pi R}{D_L} \qquad 6.11$$

Example: Jackscrew has a lever arm of 40 cm and a pitch of 5mm. If the efficiency is 30%, what force F, is required to lift a load W of 270kg?

Solution

We apply equation 6.9; IMA = 2π (0.40) / 0.005 = 502

Then we need to work out efficiency; Eff. = AMA / IMA

AMA = 0.30 x 502 =151

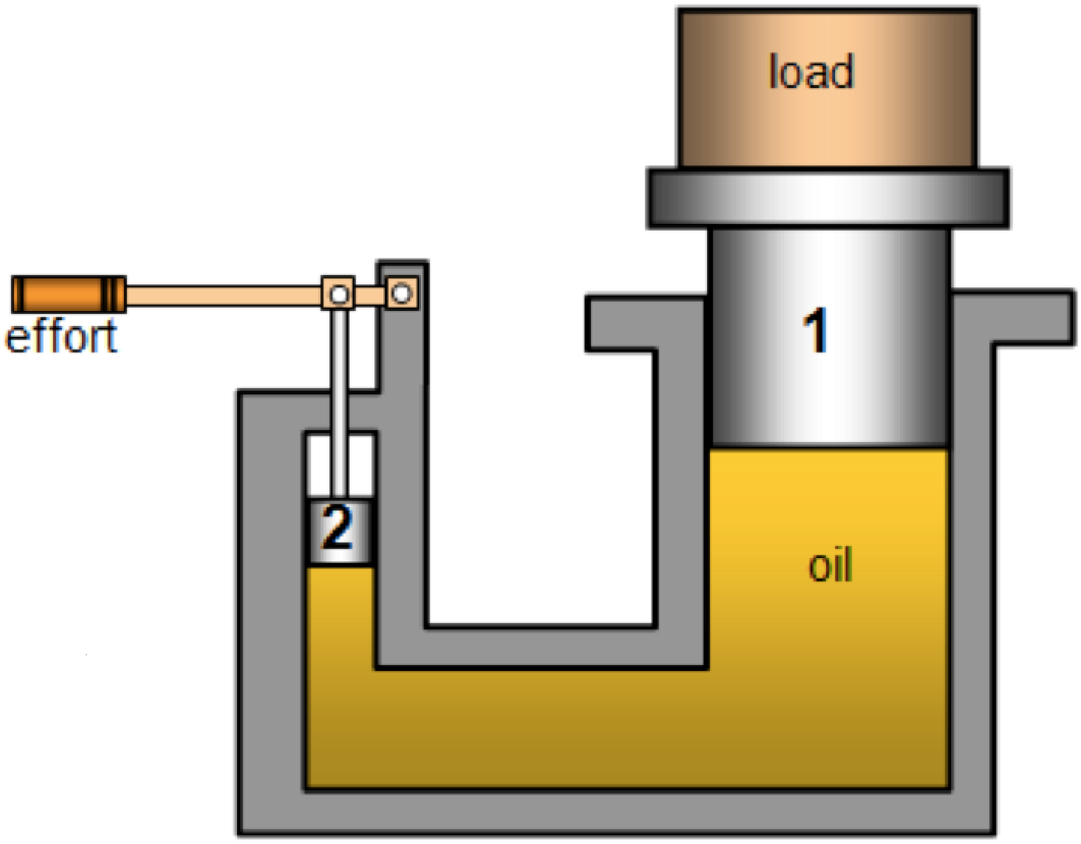
Then F = Load lifted /AMA = 270 x 9.81/ 151 = 17.5N

6.6.3 **Hydraulic Press**

The principal of liquid pressure is applied in these machines. The effort is transmitted to the load through an incompressible fluid.

Activity 6

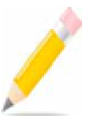
A student designed a jackscrew for pressing to tobacco bells. If the pitch of the screw is 4mm, the lever arm is 50 cm and the jackscrew is 80% efficient, find the values of the force required to press a 200kg load.



Mechanical Advantage and Velocity Ratio of a Hydraulic Press

We apply equation 6.1 to work out mechanical advantage of a hydraulic press. The ratio of the area of the piston carrying the load (1) to the area of the piston exerting the effort (2) gives us velocity ratio.

$$VR = \frac{Area..1}{Area..2} \qquad 6.12$$

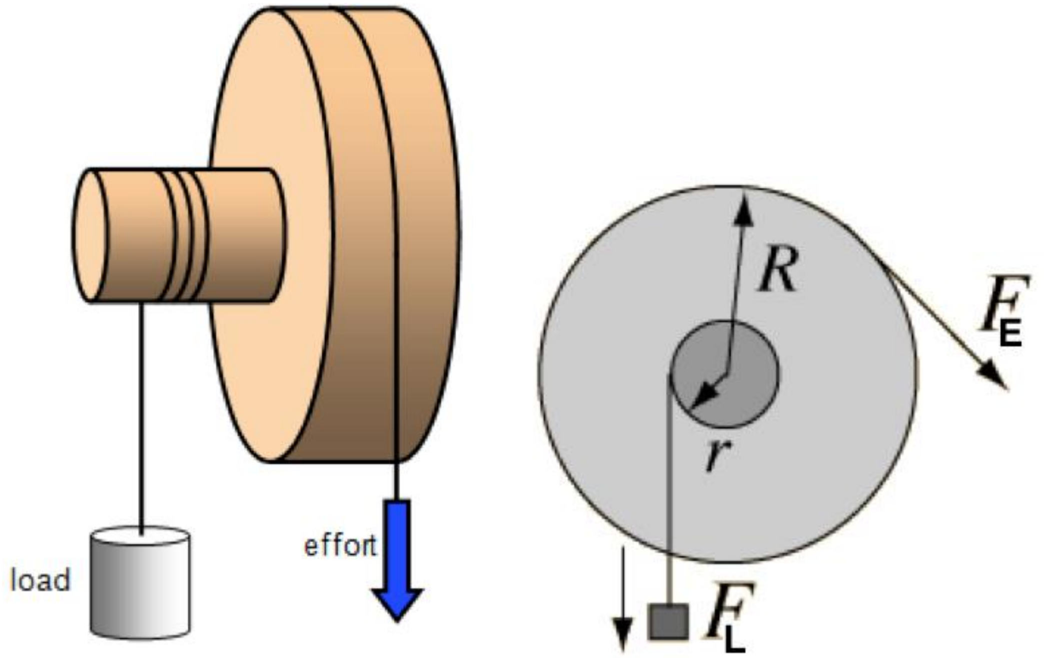


Activity 7

Calculate the ideal mechanical advantage of a hydraulic press with an effort piston of 100cm in diameter and a load piston of 300cm in diameter.

6.6.4 **Wheel and Axle**

The wheel and axle is essentially a modified lever, but it can move a load farther than a lever can. The center of the axle serves as a fulcrum. The effort is applied to the larger radius wheel and the smaller radius axle lifts the load.



Ideal Mechanical Advantage or Velocity Ratio

We define IMA for wheel and axle by taking the ratio of the radius of the wheel (R) to the radius of the axle (r). Mechanical advantage will simply be the ratio of F_L to the ratio of F_E as it is in equation 6.1.

$$VR = IMA = \frac{R}{r} \qquad 6.13$$

6.6.4.1 **Pulleys**

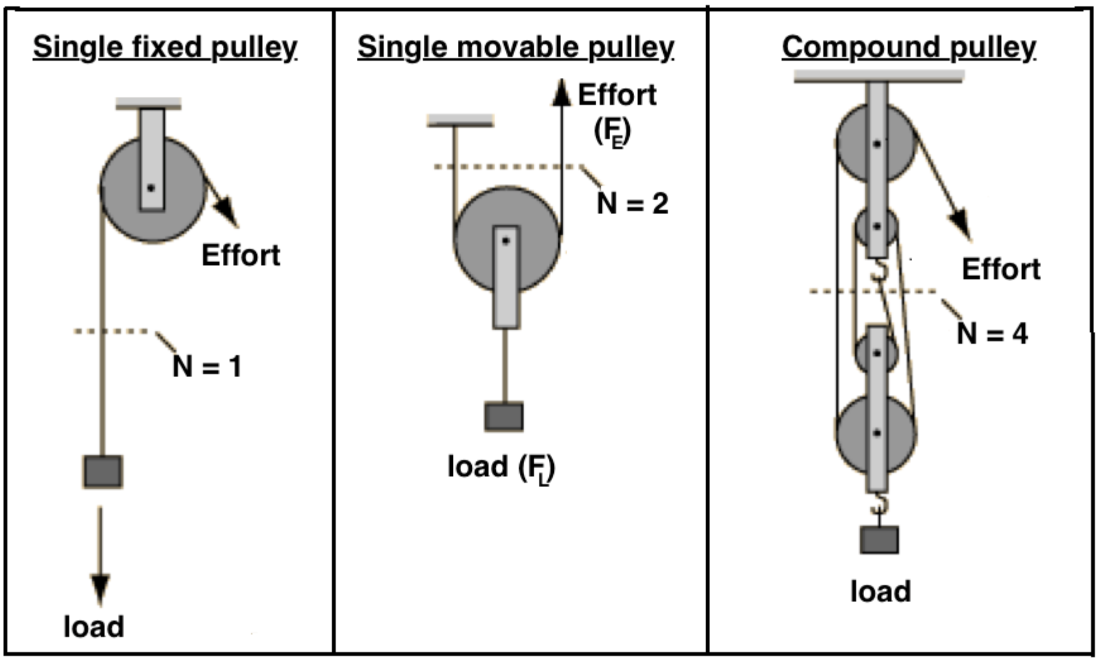
A pulley is a wheel over which a rope or belt is passed. It is also a form of the wheel and axle. Pulleys are often interconnected in order to obtain considerable mechanical advantage.

Mechanical Advantage and Velocity Ratio

Velocity ratio or ideal mechanical advantage of pulleys depends on whether the pulley is movable or not and the number of ropes supporting the load. The more the number of ropes the greater the velocity ratio. We can increase mechanical advantage of pulleys by arranging different combinations of movable and fixed pulleys. However, you should note that, increasing number of pulleys does not directly increase mechanical advantage but it increases the number of ropes supporting the load and this increases ideal mechanical advantage.

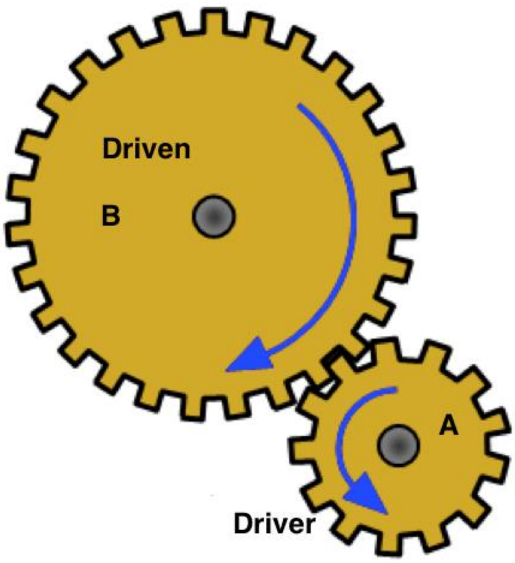
A fixed pulley has VR or IMA of 1 whereas a movable pulley has VR or IMA of 2. A compound pulley with many ropes supporting the load has a VR of greater that 2.

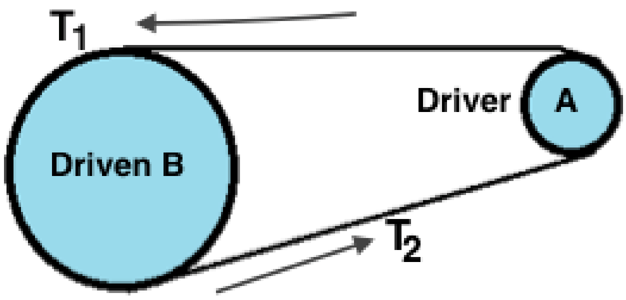
IMA = N



6.6.4.2 **Gears and belts**

When we hear the word gear, we quickly think about motor vehicles. But we have many places where gears are applied like radio cassettes, bicycles, electric motors, etc. Gears are a very widely used application of the wheel and axle.





Torque transmission

Belts and Gear drives make possible the transmission of torque from one shaft to another. Theoretically MA of a pulley system equals the ratio between the radius of the driven pulley V_{out} and that of the driving pulley V_{in}

IMA = VR = $\frac{V_{out}}{V_{in}} = \frac{d_{out}}{d_{in}} = \frac{speed..of..A}{Speed..of..B} = \frac{D_A}{D_B}$ 6.14

Power transmitted = (T_1 –T_2) x belt speed

In the case of a gear drive, since the number of teeth is proportional to its radius

IMA= VR = $\frac{V_{out}}{V_{in}} = \frac{\#..of..teeth..on..driven..gear}{\#..of..teeth..on..driven..gear}$ 6.15

If we consider the above figure, IMA is 2.

Activity 8

A bicycle is an example of gears that don't have to be touching one another to work together. This is possible because of a chain that connects the gears. This chain has links in it that fit into the teeth of the gears. What name is given to a gear that has teeth that fit into the links of a chain?

6.7 practice activity

Having discussed several categories of gears, you are now supposed to carry out an experiment and write a report as guided in Physics practical module, unit number 6.

6.8 Unit Summary

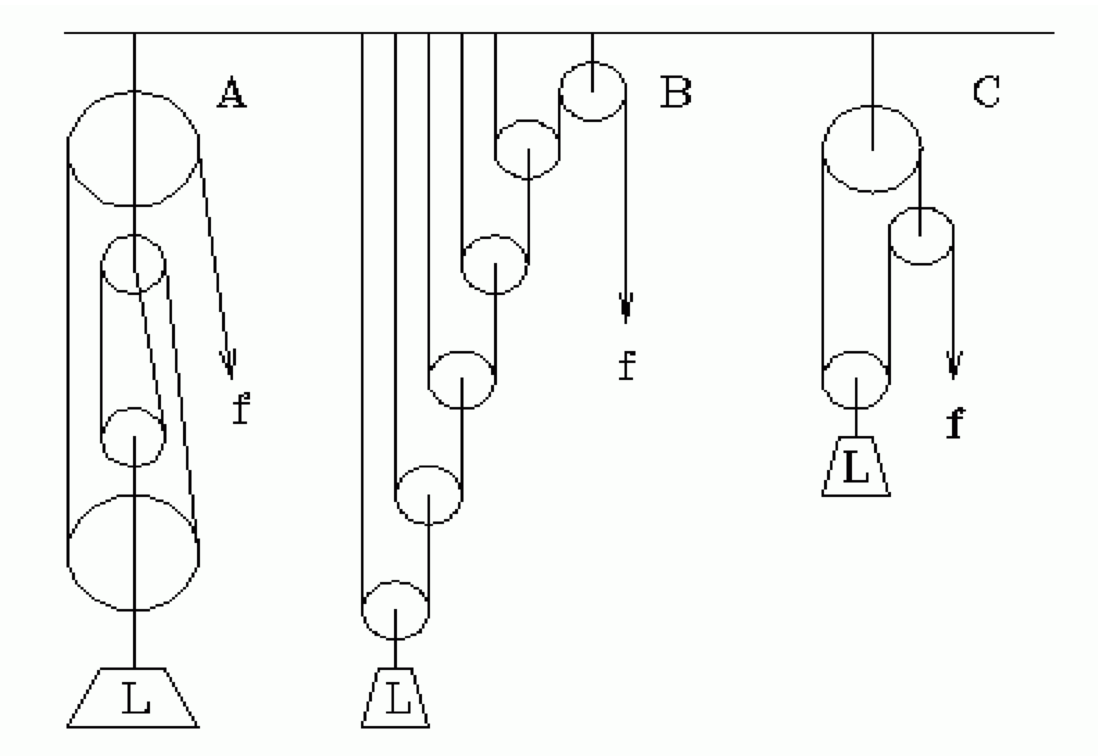
In this unit we have discussed that machines are devices that make work easier. Actual mechanical energy is the mechanical energy of a system with friction. I deal mechanical energy which is also referred to as velocity ratio is the mechanical advantage of a system without consideration of friction force. In all categories of simple machines, mechanical advantage is simply the Ratio of the load to the effort. We have also seen that ideal mechanical advantage or velocity ratio differs from one category of machines to the other depending on machines' structure and functionability. Efficiency of simple machines is given by the ratio of AMA to the IMA.

Unit Test

1. Define velocity ratio
2. What name is given to a simple machine that converts rotational motion to linear motion
3. State how many simple machines are in each of the following
    a. Wheel barrow
    b. Screw driver
    c. Stapler

d. A pair of scissors

4. A10 N force is applied to the 3m long lever. The load is placed 1m from the fulcrum. Calculate the following
   a. Effort's distance
   b. IMA
   c. AMA
   d. Efficiency
5. Define the following:
   a. Effort force.
   b. Work input.
6. Explain the relationship between effort force, effort distance, and work input.
7. In case of a fixed pulley, what is the maximum load that you can lift?
8. Give one example of a machine where friction is both an advantage and a disadvantage.
9. Evaluate systems of pulleys below and give an appropriate value of velocity ratio for each pulley, A, B and C.



Suggested Answers to unit Activities

Activity 1

Levers	Inclined planes	Wheel and axle	Hydraulic press
Broom Cooking stick	Zipper Knife Doorsteps An axe	Door knob Screwdriver	Hydraulic jack lifter

You should take note that a screwdriver can also be considered as a lever.

Activity 2

Ideal mechanical advantage is the is the mechanical advantage of the machine that does not take into account frictional losses where as actual mechanical advantage is the mechanical advantage that takes into account all frictional losses of the machine parts.

Activity 3

Answer: 4

Activity 4

Friction lowers the efficiency of a machine. Work output is always less than work input, so an actual machine cannot be 100% efficient.

Activity 6

Answer: 3.12 N

Activity 7

Answer: 9

Activity 8

Answer: Sprocket

Unit Test.

1. Velocity Ratio is the ratio of the distance moved by the load to the distance moved by the effort.
2. A screw
3. We categorize these machines as follows
   a. Wheelbarrow contains wheel and axle, a lever and an inclined plane.
   b. Screw driver contains wheel and axle and a lever
   c. Stapler comprise a lever and a wedge (inclined plane)
   d. Scissors contains a lever and a wedge
4. Here we will simply apply equations 6.1, 6.4 and 6.7
   a. 2m
   b. 2
   c. 2
   d. 100%
5. We define these terms as:
   a. The effort force is the force applied to a machine.
   b. Work input is the work done on a machine.
6. The work input of a machine is equal to the effort force times the distance over which the effort force is exerted.
7. Since a fixed pulley has a mechanical advantage of one, it will only change the direction of the force applied to it. You would be able to lift a load equal to your own weight, minus the negative effects of friction.
8. One probable example is a car jack lifter. Advantage of friction: It allows a car to be raised to a desired height without slipping. Disadvantage of friction: It reduces efficiency

GLOSSARY

Accuracy: the measure of how close the measured value of a quantity corresponds to its true value.

Actual Mechanical Advantage: The ratio of load to effort that considers friction

Basic Units: Primary units from which other units are derived

Component: x and y projections of a vector

Derived Units: Secondary units obtained through a combination of basic units

Dimension: A physical variable used to specify the behavior or nature of a particular system

Dimensions: The powers to which basic quantities must be raised to express a physical quantity

Efficiency: Is the ratio AMA to IDA or load over effort

Elasticity: The ability of a material to retain its original shape and size after bee stretched

Energy: Measure of a change imparted into a system

Equal vectors: Vectors with same magnitude, dimension and direction

Force: Product of mass and acceleration

Friction: Force that resists motion

Gears: is a toothed wheel having a special tooth shape or profile enabling it to mesh smoothly with other toothed wheels.

Gravity: A force experienced by a body due to the gravitational fields of the earth

Ideal Mechanical Advantage: A ratio of load to effort that does not consider friction

Moment: The turning effect about a point

Motion:

Parallelogram rule: Resultant of two vectors is the diagonal found after completing the vectors into a parallelogram.

Physical quantity: A quantity that can be measured

Polygon of forces: Resultant of forces is found by completing forces into a polygon

Power: The rate of dissipating energy

Precision: Degree of closeness between repeated measurements

Pressure: The ratio of force to area

Pulleys: A wheel over which a rope or belt is passed

Resolution of a vector: Is the process of getting x and y projections of a vector

Resultant: The result obtained after adding or subtracting vectors.

Scalar: A quantity with magnitude only

Strain: Deformation due to an internal state of stress

Stress: Internal resistance by a material to any tendency towards deformation

Torque: The product of a force and its perpendicular distance

Triangular rule: A resultant of two vectors is obtained by completing the vectors into a triangle

Vector: A quantity with both magnitude and direction

Velocity Ratio: The ratio of the distance moved by the effort to the distance moved by the load

Work done: Transfer of energy from one form to another

Work-Energy Theorem: Net work done in a body is equal to change in kinetic energy

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MODULE TEST

Question 1

A sprinter running a 100 meter race starts at rest, accelerates at constant acceleration with magnitude **A** for 2 seconds, and then runs at constant speed until the end.

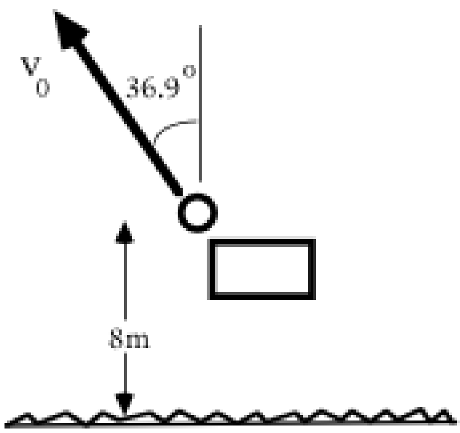
1. a) Find the position (relative to the start position) and speed of the runner at the end of the 2 seconds in terms of **A**.
2. b) Assume that the runner takes a total of 10 seconds to run the 100 meters. Find the value of the acceleration **A**. You can leave your answer in terms of a fraction but clearly indicate the units.

Question 2

A rock is thrown upward from a bridge at an initial height of 8 meters above the water at an initial speed of **v** and an angle of 36.9 ^{o} from the vertical as shown. Use g=10 m/s^2 to solve this problem.

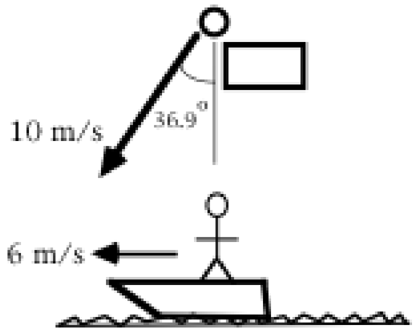
a. Write a set of equations for the horizontal and vertical positions and velocities of the rock as a function of time. Clearly indicate on your drawing your choice of axes and what point you are using as your origin.

b. The rock reaches its highest point in 2 seconds. How high is the rock the water at that instant? (Hint: First you need to find $\mathbf{v_0}$.)



Question 3

A rock is thrown downward from a bridge at an initial speed of 10 m/s and an angle of 36.9^{o} from the vertical as shown. At the o same instant a boat is passing under the bridge traveling 6 m/s in the direction shown. See note on formula sheet about the values of trigonometric functions for this angle.



a. Find the vertical and horizontal components of the initial velocity of the rock as seen by a person on the bridge. Clearly indicate on your drawing your choice of axes.
b. Find the vertical and horizontal components of the initial velocity of the rock as seen by the person on the boat. Clearly indicate on your drawing your choice of axes.
c. Draw a clear vector diagram showing how to relate the velocity the rock appears to be moving as seen from the bridge, the velocity the rock appears to be moving as seen by the person in the boat, and the velocity of the boat with respect to the bridge.

Question 4

a. A micrometer screw gauge is used to measure the diameter of a small uniform steel sphere. The micrometer reading is 5.00 mm ± 0.01 mm. What will be the percentage uncertainty in a calculation of the volume of the sphere?

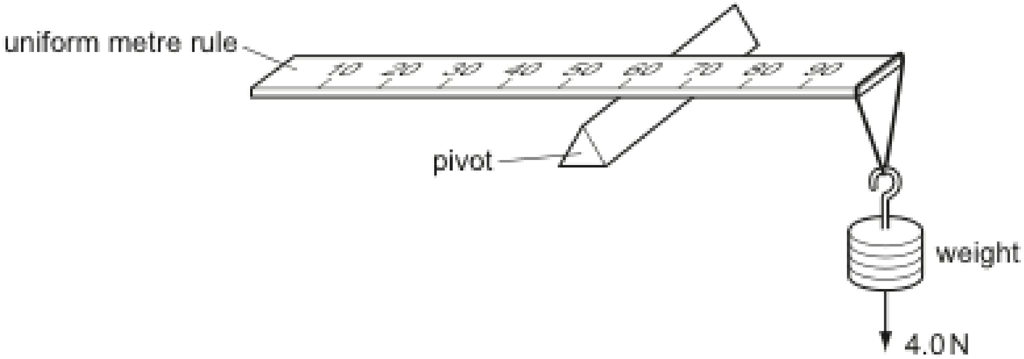
b. The spring constant k of a coiled wire spring is given by the equation

$$K = Gr^4\ 4nR^3$$

where r is the radius of the wire, n is the number of turns of wire and R is the radius of each of the turns of wire. The quantity G depends on the material from which the wire is made. Calculate a suitable unit for G.

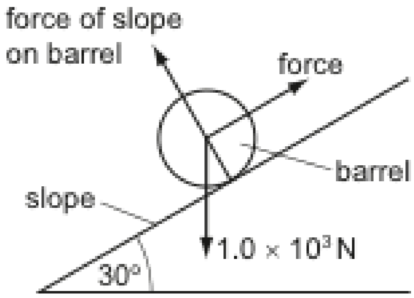
Problem 5

A uniform metre rule of weight 2.0N is pivoted at the 60cm mark. A 4.0N weight is suspended from one end, causing the rule to rotate about the pivot. At the instant when the rule is horizontal, what is the resultant turning moment about the pivot?



Question 6

The diagram shows a barrel of weight 1.0×10^3 N on a frictionless slope inclined at 30° to the horizontal.



A force is applied to the barrel to move it up the slope at constant speed. The force is parallel to the slope. What is the work done in moving the barrel a distance of 5.0 m up the slope?